A TRUNCATED TREE SEARCH APPROACH FOR GENERALIZED GROUPING

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Abstract

Part machine grouping is considered as backbone of cellular manufacturing system. A review of the literature suggests a need of more realistic approach to machine grouping problems. Many of the earlier works have neglected some important considerations. In the present work, an attempt has been made to enlarge the scope of the grouping problem by incorporating more parameters to bring the problem closer to the real life industrial problem, such as processing time required by parts for each process, production volume of parts, operation sequence of parts, alternate process plans for each parts, number and types of machines, upper limit on cell size, material handling costs related to intercell and intracell moves, cost of processing of parts. Objective is to minimize the sum of processing costs and material handling costs related to intercell and intracell movements. The resulting grouping problem is formulated as a (0-1) integer programming problem.

Here objective function is non linear and formulation is NP- hard. Number of computations required in solving this type of problem by conventional mathematical programming will be very high. So, truncated tree search based heuristic is proposed to solve the resulting grouping problem.

Complexity of the proposed heuristic comes out $O\left(M^3\{wL^3NR_p + w + wR_p{}^2\}\right)$ where M is total number of machines, N is total number of parts, R_p is total number of number process plans for a part, L is the average number of copy of machines of one type and w is the truncating parameter.

Various type of problems have been solved and effect of the problem size, the routing flexibility and the truncating parameter on the solution, the user time required and the value of objective function are observed and discussed which validate the proposed solution procedure. It is also observed that proposed solution procedure is able to solve the problem in reasonable lesser time. The truncating parameter, the most important characteristic of this solution methodology which determines the quality of solutions and user time should be assigned in such a manner that a balance between these two will be reached at.

CERTIFICATE

It is certified that the work contained in the thesis entitled **"A Truncated Tree Search Approach for Generalized Grouping"** by Suman Raaj has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Chapter 1

GROUPING PROBLEM : AN INTRODUCTION

1.1 Cellular Manufacturing Systems

For improving productivity and flexibility, cellular manufacturing systems is widely accepted as one of the major application of group technology in manufacturing. Part machine grouping is considered as backbone of cellular manufacturing system. It mainly involves identification of part families, formation of machine cells and assignment of part families to respective machine cells. In part machine grouping problem, the main objective is to obtain grouping solution containing perfect groups in which parts do not have to move from one machine cell to another machine cell for processing. But there are many factors viz. processing time required by parts for processing, production volume of parts, number and size of machine cells, costs considerations, that tend to drive grouping solution far from perfect.

In simple grouping problem with single process plan for each part, it is not always possible to get perfect groups. There is a possibility of getting such a situation, in which a single machine is required by parts belonging to different part families. These machines are referred to as bottleneck machines. For improving these type of solutions, we can merge those groups of parts/machines, or we can employ additional copies of machines and assign them to each or some of the concerned groups depending on the economic consideration.

In flexible manufacturing systems environment, where parts have more than one process plans, the grouping problem is referred to as generalized grouping and in this case it is possible to form better groups as compared to the simple grouping.

1.2 Literature Review

There are many techniques for solving group formation problem. An overview of some grouping techniques is presented in table 1.3. These techniques are classified on the basis of input(s), objective(s) and basic approach(es). The literature review is organized and grouped according to the following scheme : type of input. parameters. The types of inputs are given in table 1.1 whereas the parameters are listed in table 1.2.

Table 1.1 : Inputs required for the grouping methods

Digit	Interpretation of the digit regarding input
1	Binary Matrix
2	Non-binary Matrix without Operation Sequence
3	Non-binary Matrix with Operation Sequence

Table 1.2 : Other parameters required for grouping methods

Digit	Interpretation of this digit regarding required input
0	No additional information used
1	Number of Cells
2	Cell Size
3	Copy of Machines
4	Capacity of Machines
5	Production Volume of Parts
6	Processing time required by parts on a machines
7	Part Routings
8	Cost of Machines and/or Machining
9	Material Handling Costs

Various grouping techniques are analyzed on the basis of input(s), objective(s) and basic approach(es).

1.2.1 According to Input(s)

In most of the grouping techniques available for cell formation, primary input is part machine incidence (0-1) binary matrix. This matrix indicates whether a part requires a machine or not. These methods do not consider production volume of parts, processing time required by parts for processing or other practical aspects, so they are not very appropriate for practical environment.

Some of the grouping techniques take input as a combination of following input parameters:

- Processing time required by parts for processing
- Production volume of parts
- Processing capacity of machines
- Number and size of groups.

These methods are better than the methods which consider only binary matrix. But these methods ignore other practical aspects *e.g.* material handling costs, machine duplication costs, costs of processing, flexibility in routing which are very important in FMS environment.

Modern grouping techniques consider a combination of processing time required by parts for processing, production volume of parts, operation sequence of parts, material handling costs, processing capacity of machines, costs of processing, number and types of machines, number and size of groups.

In further development, a term `alternate routing' became very important which is associated with FMS environment. Some of the modern techniques consider alternate process plans for parts in addition to the input combinations for the techniques developed earlier. Most of the techniques do not consider all practical aspects together. So there is still a possibility to develop techniques which consider more aspects and produce more adequate results to real industrial environment.

1.2.2 According to Objective(s)

Most of the grouping techniques have objective to form part machine cluster or to minimize dissimilarity among parts/machines belonging to same group or to minimize bottleneck machines and exceptional parts.

In modern grouping techniques, objective is changed to minimization of intercell moves. But in practice, intracell moves are also as important as intercell moves. Backtracking cost, subcontracting cost of parts and machine duplication cost are also equally important.

So in further development, some of the techniques have objective to minimize total moves and some of them have objective to minimize total costs. Total costs/moves are combination of all costs/ moves stated above. These classes of objectives i.e. total moves or total costs are closer to real problem.

1.2.3 According to Basic Approach(es)

Generally, grouping problem is NP hard and these approaches are categorized in two types. One in which part families and machine cells are formed hierarchically and the other in which both are formed simultaneously. In general, mathematical programming approach can accommodate much more practical aspects than the heuristics. So, many grouping techniques have the basic approach of mathematical programming. Generally, grouping techniques having this approach forms part families and machine cells hierarchically. Since grouping problem is NP hard, so solving these problems by mathematical programming approach is computationally very complex. To overcome the computational complexity of mathematical

programming and to bring flexibility in solutions, efficient heuristics are used. Most of the grouping techniques use similarity coefficient based heuristics as basic approach which provides good solutions not necessarily optimal, in lesser computation. Some of techniques use graph formulation as basic approach. Simulated annealing, network flow algorithm, genetic algorithm, truncated tree search algorithm, fuzzy clustering, neural network based clustering, Lagarangian relaxation based heuristics, requirement based clustering heuristics also have been used as basic approach for grouping techniques. Some techniques use a combination of above heuristics.

1.3 Present Work

In the present study, our approach is to solve grouping problem and to form machine cells by considering the following factors :

- Processing time required by parts for each process
- Production volume of parts
- Operation sequence of parts
- Alternate process plans for each parts
- Number and types of machines
- Upper limit on cell size
- Material handling costs related to intercell and intracell moves
- Cost of processing of parts.

Objective is to minimize the sum of processing costs and material handling costs related to intercell and intracell movements. More specifically, the aim is to form such type of machine cells in which material handling costs is minimum. But processing costs are equally important. So for machine cell formation, a different objective, sum of the processing costs and the material handling costs related with movements, is taken.

Here objective function is non linear and formulation is NP hard. Number of computations required in solving this type of problem by conventional mathematical programming will be very high. Non conventional techniques i.e. simulated annealing, genetic algorithm, tabu search may be used for solving this problem. These techniques are good enough for producing a result but these techniques are also computationally very complex. Heuristics like neural network or fuzzy clustering takes less time in computation but quality of solution is not very good. Truncated tree search heuristic is capable to produce good result as

comparable to optimal solution in less computation [Chang et al, 1996]]. With this in view, a truncated tree search based heuristic is proposed to solve the resulting grouping problem.

1.4 Organization of Thesis

Entire thesis is divided into four chapters.

First chapter introduces the concept of cellular manufacturing system and reviews the available literature in the area.

Second chapter deals with problem environment, problem formulation, solution methodology, proposed algorithm and complexity analysis of proposed algorithm.

Third chapter discusses the results obtained by solving a number of different varieties of problems.

Fourth chapter presents the conclusions and scope for further work in this area.

Certain supporting results and explanations are presented in appendices.

Table 1.3 : Classification of grouping methods
(on the basis of their inputs required, basic approach, objective and procedure outline)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 1.0 : Input - Binary Matrix, No additional information			
Ballakur and Steudel [1987]	Graph Formulation	To maximize the within cell utilization	1. Generate a bi-partite graph of machines and parts 2. Select a key machine or part according to prespecified criterion 3. Apply depth first search on the graph and select candidate cell. 4. Eliminate the vertices corresponding to parts and machines. 5. Machines and parts are assigned to cells according to cell utilization.
Vishwanathan [1996]	Integer Programming	Part machine clustering	By using a different measure of similarity coefficient an integer programming problem is formulated and solved by standard packages.
Boe and Cheng [1991]	Integer Programming Problem Solved by Heuristic	To maximize the sum of closeness measure between machines	1. A Machine which has the largest sum all non-diagonal elements of matrix serves as seed and occupies the first row of final matrix. A Machine is selected for rows if it is closest to the machine selected earlier. 2. A Part having largest string of non-breaking 1's is assigned to left most column subsequently.
Lee and Diaz [1993]	Network Flow	Part machine clustering	1. Machine network is created and distance parameter is calculated. 2. Now network flow problem is formulated and solved by standard packages.
Cho [1993]	Neural Network	Part machine clustering	1. After taking the input train the network. 2. Test the classification. 3. Display clustering and evaluate the performance.
Kaparthi and Suresh [1992]	Neural Network	Part machine clustering	Every part is considered as n- dimensional vector and Carpenter Grossberg network is applied to obtain part families.
Chen and Chang [1995]	Neural Network Followed by Heuristic	To reduce bottleneck condition	1. Obtain basic solution by using ART1. 2. We identify those machines and parts having more 1's outside the cell than inside the cell. They are set aside. Now machines are arranged according to decreasing order of 1's within cell. At last, machines and parts which are set aside, are presented to cells according to previous step. 3. Machines are assigned to their cell which has the largest number of parts processed by that machine.
King [1980]	Heuristic	Part machine clustering	Rows entries are read as binary words and they are ranked in reducing order and this is done for column also. After this, a matrix having block diagonal structure is obtained.
Srinivasan [1994]	Heuristic	Part machine clustering	1. After calculating distance function, minimum spanning tree is constructed. 2. Parts and machines are clustered by using GRAPHICS algorithm until machine groups are equal to part families.

Contd.

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 1.0 : Input - Binary Matrix (Contd.)			
Chen and Irani [1993]	Heuristic Followed by TSP Problem and Steepest Descent Method	Part machine clustering	1. Clustering is done by minimum spanning tree. 2. Linearization is done by Traveling Salesman Problem. 3. Optimization is done by Steepest Descent Method.
Chandra-shekharan and Rajagopalan [1987]	Similarity Coefficient	Part machine clustering	1. At first clustering is done by non- hierarchical method, such as by generating some artificial seed and using them as fixed seed point. 2. Diagonalization is done. 3. From resulting clustered matrix, ideal seeds are identified and clustering is done by those seeds
Chow and Hawaleshka [1993]	Similarity Coefficient	Part machine clustering	1. Commonality scores are computed for each pair of machines. 2. A pair of machines having highest score is grouped or combined. 3. This group is replaced by a new machine unit. 4. These steps are repeated until all machines are grouped.
Mukhopadhyay et al [1994]	Similarity Coefficient	Part machine clustering	1. Calculate strength coefficient between machines using similarity coefficient concepts. 2. Machine selection is done according to these coefficient and simultaneously part selection is also done.
Mukhopadhyay et al [1995]	Similarity Coefficient	Part machine clustering	1. Machine is read as n-dimensional vector and cosine of a machine I and other machines are calculated. 2. Machine having maximum cosine value with machine I is placed next to machine I. Now I is increased by 1 and this process is repeated till all machines are placed. 3. Same procedure is repeated for parts. 4. Depending on the decision rules based on the threshold value, groups are formed.
Mukhopadhyay et al [1997]	Similarity Coefficient	Part machine clustering	1. One iteration is made Similarity coefficient between machines pair and parts pair are calculated. 2. In stage 1, machines are grouped and in stage 2 parts are grouped. or In stage 1, parts are grouped and in stage 2 machines are grouped. or for machine grouping in stage 1, after that one iteration is made for part grouping in stage 2. Again one iteration is made for machine grouping and after that one for part grouping and so on until convergence occurs.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 1.0 : Input - Binary Matrix (Contd.)			
McAuley [1972]	Similarity Coefficient	Part machine clustering	1. Similarity coefficients for each ,pair of machines are calculated. 2. A Dendogram is drawn and for a desired value of threshold value machine groupies are formed.
Rogers and Shafers [1993]	Similarity Coefficient	Part machine clustering	A New measure of similarity coefficient is introduced and calculated for each pair of machines and McAuley method is used for grouping machines.
Srinivasan et al [1990]	Similarity Coefficient Followed by Assignment Problem	Part machine clustering	1. Similarity matrix is computed between machines and it is solved by assignment problem concept. Closed loops are identified which constitutes a group of machines. 2. Part families are assigned to minimize the exceptional elements. 3. Machine groups are merged which have no part families. 4. Merging is done when number of voids created by this action is less than number of exceptional elements eliminated.
Veeramani and Mani [1996]	Vertex Tree Graphic Matrix	Part machine clustering	1. Obtain initial solution by Tarjan's algorithm. If it is successful then stop. 2. Check whether matrix is Vertex Tree- graphic or not. For this a minimal d-set matrix is found out followed by building of decomposition of tree followed by finding out for critical decomposer at each node and check for mergeability. 3. Find out all edges cut followed by finding out of all minimal d- set. 4. Find out the best assignment of columns of each minimal d-set.
Table 1.1 : Input - Binary Matrix, Number of Cells			
Boctor [1996]	Integer Programming Solved by Simulated Annealing	To minimize the number of exceptional elements	Formulation is done as integer programming problem and solved by simulated annealing heuristic.
Chu and Hayya [1991]	Fuzzy Clustering Approach	Part machine clustering	1. Fuzziness matrix is evaluated by Picard iteration process. In the final fuzziness matrix, part is assigned to cell with maximum value of fuzziness. 2. Machine clustering is done by examining the final cluster means.
Table 1.2 : Input - Binary Matrix, Cell Size			
Malakooti and Yang [1995]	Neural Network	To minimize total dissimilarity among machines belonging to same cell	1. Cell centers are found out by neural network clustering analysis. 2. Clustering of machines are based on the distance between machine vectors and centers considering lower and upper bound of cell size. Clustering of parts are done by using same procedure.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 1.2;1 : Input - Binary Matrix, Number of Cells, Cell Size			
Wang and Roze [1997]	Integer Programming	To maximize the similarity among parts/ machines belonging to same cell	An integer programming problem is formulated and solved by any standard packages.
Murthy and Srinivasan [1995]	Integer Programming Problem Solved by Package or Simulated Annealing or Heuristic	To minimize exceptional elements	1. An integer programming problem is formulated. 2. If number of variables are less than solve it by some standard packages. If number of variables are large then solve it by simulated annealing heuristic. If number of variables are very large then solve it by a heuristic. This heuristic includes Vogel's approximation method for solving transportation problem.
Chen et al [1995]	Simulated Annealing	To minimize intercell moves	Initially approximately equal number of machines are assigned to machine cells arbitrarily. Randomly one machine is picked from one cell and put it to another cell. After that total moves are calculated. Hence, simulated annealing is applied to solve this problem.
Table 1.3 : Input - Binary Matrix, Copy of Machines			
Chang et al [1996]	Quadratic Integer Programming Problem Solved by Heuristic	To minimize total distance	1. Quadratic integer programming problem is formulated and distance matrix is computed. 2. Initial node is placed. A node is branched by assigning an unassigned machine to machine cells one by one, one at a time and its successor nodes are generated. 3. A node is pruned if it violates constraints. For all feasible nodes, objective function is calculated. Nodes having a specific number of better objective function are taken and other nodes are pruned. These procedures are repeated until all machines are assigned.
Table 1.3;2;1 : Input - Binary Matrix, Copy of Machines, Number of Cells, Cell Size			
Vishwanathan [1995]	Quadratic Integer Programming Problem Solved by Heuristic	To minimize intercell moves	1. Let k be the maximum number of cells. Assign $(k-1)$ machines and parts to $(k-1)$ cells and rest of them to kth cell. 2. Now starting from first part, each part is moved and assigned to that cell which gives maximum reduction in objective function for movement due to that assignment. 3. This procedure is repeated for machines. 4. If both of the above procedures are not able to improve the objective function then both parts and machines are moved simultaneously from one cell to another cell. 5. Absorb the empty cells. then machine and parts are moved from the cells having machines more than upper limit of cell size to cell for which smallest increase in objective function is occurred by movement.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 1.7;1 : Input - Binary Matrix, Part Routings, Number of Cells			
Kusiak [1987]	Integer Programming	To maximize the similarity among parts/machines belonging to same cell	An integer programming problem is formulated and solved by any standard packages.
Kusiak and Cho [1992]	Similarity Coefficient	Part machine clustering	1. Similarity coefficient between process plans of parts are calculated and transition graph is constructed. 2. Maximum clique is determined and rows and columns are removed from initial similarity coefficient matrix corresponding to process plan and this clique is branched. 3. A Clique branch node which produces larger number of process plan family then current solution, is fathomed. 4. Stop if current solution is acceptable else above procedures are repeated.
Table 1.7;3 : Input - Binary Matrix, Part Routings, Copy of Machines			
Adil and Rajamani [1996]	Integer Programming Problem Solved by Simulated Annealing	To minimize the weighted sum of exceptional elements and voids	An integer programming problem is formulated and solved by simulated annealing heuristic.
Table 1.7;6;2 : Input - Binary Matrix, Part Routings, Production Volume of Parts, Cell Size			
Song and Hitomi [1993]	Graph Formulation Followed by Mathematical Programming	To minimize intercell moves	1. QAP formulation is done for maximizing the number of parts in a cell. 2. Graph formulation is done with machines as node and number of parts common to both machines as arcs joining those machines. 3. Formulations are solved by Lagrangian relaxation based heuristic. 4. Global optimal solution is obtained by branch and bound algorithm.
Table 1.8;7;6;3 : Input - Binary Matrix, Cost of Machines, Part Routings, Production Volume of Parts, Copy of Machines			
Rajamani et al [1994]	Integer Programming	To minimize capital investment	There are three integer programming problem models are formulated for same problem. 1. First model assign machines to parts using any clustering algorithm. 2. Second model assign machine to part families. Part families are formed by using part attributes. 3. Third model obtain part families and machine cells simultaneously considering demand of parts and resource constraints. All models are solved by any standard packages.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 2.6;5 : Input - Production Volume of Parts, Processing Times Required by Parts on Machines			
Gunasingh and Lashkari [1989]	Integer Programming Problem Solved by Heuristic	Machine Cell Formation	1. First integer programming problem model maximize the compatibility index between parts and machines and seeks a trade off between cost of allocating the machine and cost of intercell moves. 2. Second model is same as first but having extra constraints. It is solved by heuristic suggested by Dutta [1986].
Vohra et al [1990]	Network Flow	To minimize total machining time during intercell movement	Network model is created. After that graph partitioning is done by modified Gomory Hu algorithm to obtain minimum cut.
Mosier [1989]	Similarity Coefficient	Part machine clustering	Similarity coefficient matrix is calculated and clustering is done by any of the clustering technique.
Mosier [1989]	Similarity Coefficient	Part machine clustering	Same as previous but differ in measurement of similarity coefficient.
Mosier [1989]	Similarity Coefficient	Part Machine Clustering	Same as previous but differ in measurement of similarity coefficient.
Table 2.6;5;4;2;1 : Input - Production Volume of Parts, Processing Times Required by Parts on Machines, Capacity of Machines, Cell Size, Number of Cell			
Boctor [1996]	Integer Programming Problem Solved by Simulated Annealing	To minimize intercell moves	An integer programming problem is formulated and solved by simulated annealing heuristic.
Delvalle at ai [1994]	Heuristic	To minimize intercell moves	1. Cellular representation is done by selection of machines equal to number of cells in such a way that first machine should be having maximum load, second should be the most different from first and third should be the most different from first and second and so on. 2. Now machine having maximum load among rest of the machines, is selected and assigned to that cell in which lowest number of intercell movement exists. This process is repeated until all machine are assigned. 3. Take the machine assigned first in previous stage and assigned it permanently to that cell having minimum intercell moves corresponding to that machine. Take and assign other machines according to order in which machines are assigned in previous stage.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 2.6;5;4;2;1 : Input - Production Volume of Parts, Processing Times Required by Parts on Machines, Capacity of Machines, Cell Size, Number of Cell			
Okogbaa et al [1992]	Heuristic	To minimize intercell moves	1. Select seed machine which processes maximum parts. 2. Select another machine which has minimum intercell moves with selected machine. 3. Assign the machine to the cells formed by seed machine on the basis of ratio of intercell moves between machine and cell, and total intercell moves. These steps are repeated until all machine are assigned. 4. Obtain optimal assignment by reassigning the machines if this assignment reduces intercell moves in the system.
Table 2.6;5;4;3;2 : Input - Production Volume of Parts, Processing Times Required by Parts on Machines, Capacity of Machines, Copy of Machines, Cell Size			
Suresh et al [1995]	Fuzzy ART Followed by Goal Programming and Integer Programming	To maximize the cell utilization with minimum purchage of machines	1. Clustering is done by neural network technique to identify part family and machine cells followed by rescanning to identify alternate family and adjustment of parameter values. 2. An interactive goal programming model is formulated and solved to maximize complete cell manufacturing with minimum purchage of machines followed by reallocation of parts and machines. 3. An integer programming model is formulated and solved to minimize intercell moves.
Table 2.7;6;5;4 : Input - Part Routings, Production Volume of Parts, Processing Times Required by Parts on Machines, Capacity of Machines			
Gupta [1993]	Similarity Coefficient	Part machine clustering	1. Determination of usage factors of different routes considering demand and capacity constraints using MANUPLAN. 2. Calculation of similarity coefficient between machines and clustering is done by any technique (preferably McAuley's).
Table 2.8;6;5;4 : Input - Cost of Machines, Production Volume of Parts, Processing Times Required by Parts on Machines, Capacity of Machines			
Rajamani et al [1992]	Integer Programming	To optimize the benefits of job shops and flow shops	An integer programming problem is formulated and solved by simulated annealing heuristic.
Amirahmadi and Choobineh [1996]	Heuristic Followed by Integer Programming	To minimize total cost	1. Set of bottleneck machines and set of exceptional parts are identified by any clustering heuristic using binary matrix as input. 2. An integer programming problem is formulated and solved to minimize total costs *i.e.* subcontracting cost of exceptional parts, purchasing cost of bottleneck machines and intercell movement cost.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 2.8;6;5;4 : Input - Cost of Machines, Production Volume of Parts, Processing Times Required by Parts on Machines, Capacity of Machines			
Ho and Moodie [1996]	Similarity Coefficient Followed by Mixed Integer Programming	To minimize total cost	1. Operation similarity coefficient is calculated and parts are grouped together according to similarity coefficient. 2. A mixed integer programming model is formulated and solved to allocate the machines.
Table 2.8;6;5;4;2;1 : Input - Cost of Machines, Production Volume of Parts, Processing Time Required by Parts on Machines, Capacity of Machines, Cell Size, Number of Cells			
Heragu and Gupta [1994]	Heuristic.	Part machine clustering	1. Identification of machine cells and part families is done considering safety and technology constraints. 2. Cell size constraints are fulfilled by duplicating machines appropriately. 3. Constraint related with number of cells, are fulfilled by merging the cells. 4. Duplicate machines are eliminated to reduce intercell moves.
Table 2.8;7;6;5 : Input - Cost of Machines, Part Routings, Production Volume of Parts, Processing Times Required by Parts on Machines			
Balasubramanian et al [1993]	Heuristic	To minimize the total cost of material handling and machine utilization	1. Take the machine required for every part as a cell. 2. Obtain additional cell arrangement using King's method. 3. Compute moves between each cell arrangement and component in a similarity coefficient matrix. 4. Select the cell from the similarity matrix on the basis of total cost using covering algorithm.
Liao [1994]	Integer Programming Followed by Neural Network	To eliminate the load imbalance problem for minimizing operation and handling cost	1. Selection of part routings is done for minimizing the operating cost considering production volume and machine capacity by integer programming problem model. 2. Solution of resulting binary matrix is obtained by ART based neural network model to form desired number of machine cell. 3. Selection of the optimum layout is done by calculating the material handling cost for cell design obtained from previous stage using STORM package.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 2.8;7;6;5 : Input - Cost of Machines, Part Routings, Production Volume of Parts, Processing Times Required by Parts on Machines, Capacity of Machine (Contd.)			
Rajamani et al [1996]	Mixed Integer Programming Problem Solved by Simplex and Branch & Bound Technique	To minimize the sum of operating and handling cost	1. A Mixed integer programming problem is formulated. 2. It is solved by simplex technique after dropping integrally constraints of variables. 3. Further optimal solution is obtained by branch and bound algorithm after solving this problem with integrity constraints of variables.
Logendran et al [1994]	Integer Programming Problem Solved by Tabu Search Heuristic	To minimize total annual cost	An integer programming problem is formulated and solved by tabu search heuristic.
Table 2.9;8;6;5;4;3;1 : Input - Cost of Material Handling, Cost of Machines, Production Volume of Parts, Processing Times Required by Parts on Machines, Capacity of Machines, Copy of Machines, Number of Cells			
Boctor [1996]	Integer Programming Problem Solved by Simulated Annealing	To minimize acquisition cost of additional machines and intercell movement cost	An integer programming problem is formulated and solved by simulated annealing heuristic.
Sofianopoulou [1997]	Integer Programming Problem Solved by Simulated Annealing	To minimize intercell moves	An integer programming problem is formulated and solved by simulated annealing heuristic.
Table 3.2;1 : Input - Operation Sequence, Number of Cells, Cell Size			
Harahalkis et al [1990]	Heuristic	To minimize intercell moves	1. Start with placing each machine in each cell and intercell traffic is calculated between cells. If it is zero then stop. 2. Two cells having maximum intercell movement with each other are merged to one cell subjected to cell size constraint. 3. If feasible merging is not possible then stop. 4. After these three procedures, solution obtained is taken. Each machine is taken as entity and is assigned to that cell with which maximum traffic is occurred.
Choobineh [1988]	Similarity Coefficient	Part machine clustering	A New measure of similarity coefficient is introduced and calculated for each pair of machines and McAuley method is used for grouping machines.
De Witte [1980]	Similarity Coefficient	Part machine clustering	A New measure of similarity coefficient is introduced and calculated for each pair of machines and McAuley method is used for grouping machines.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 3.2;1 : Input - Operation Sequence, Number of Cells, Cell Size (Contd.)			
Shiko [1992]	Similarity Coefficient	Part machine clustering	At first the similarity coefficient is calculated for each pair of process plans. The process plans are combined according to similarity coefficient to form standard process plan until standard process plan formation is not possible. When formation of standard process plans is not possible then these standard process plans are taken as final grouping.
Tam [1990]	Similarity Coefficient	Part machine clustering	A New measure of similarity coefficient is introduced and calculated for each pair of machines and McAuley method is used for grouping machines.
Table 3.6;5;2 : Input - Operation Sequence, Production Volume of Parts, Processing Time Required by Parts on Machines, Cell Size			
Wu et al [1993]	Graph Formulation	To minimize intercell moves	Graph formulation is done. Algorithm 1 1. Obtain a cut set graph by Gomory Hu algorithm. 2. Cut the tree at the minimum intercell move arc until number of nodes in any tree is greater than size of cell. 3. Take each tree as a machine cell. Algorithm 2 Take two seed node with minimum and maximum intercell moves for applying Gomery Hu algorithm to obtain cut sets and apply algorithm 1.
Table 3.6;5;4 : Input - Operation Sequence, Production Volume of Parts, Processing Time Required by Parts on Machines, Capacity of Machines			
Gupta and Seifoddinni [1990]	Similarity Coefficient	Part machine clustering	A New measure of similarity coefficient is introduced and calculated for each pair of machines and McAuley method is used for grouping machines.
Table 3.7;6;5 : Input - Operation Sequence, Part Routings, Production Volume of Parts, Processing Times Required by Parts on Machines			
Nagi et al [1990]	Linear Programming Followed by Heuristic	To minimize intercell moves	1. A Linear programming problem is formulated and solved for the selection of routes. 2. Use of intercell movement minimization heuristic for grouping in the presence of operation sequence.
Table 3.9;3;2 : Input - Operation Sequence, Cost of Material Handling, Copy of Machines, Number of Cells			
Shanker and Agrawal [1997]	Hierarchical Approach - Traveling Salesman Problem Followed by Heuristic and Transportation Problem	To maximize the association between parts and machines and to minimize intercell moves	Part families are determined by determining part chain which is done by formulating and solving a traveling salesman problem. After that part family search is done by a heuristic based on bond strength. Machine assignment is done by formulating and solving a transportation problem.

(Contd.)

Table 1.3 : Classification of grouping methods (Contd.)

Author(s)	Basic Approach	Objective	Procedure Outline
Table 3.9;3;2 : Input - Operation Sequence, Cost of Material Handling, Copy of Machines, Number of Cells (Contd.)			
Shanker and Agrawal [1997]	Non- Hierarchical Approach -- Linear Programming or Requirement Based Clustering or Langarangian Relaxation Based Heuristic	To maximize the association between parts and machines and to minimize intercell movement cost	If perfect grouping is possible then an integer programming problem is formulated and solved. If perfect grouping is not possible then A Sequential approach requirement based clusterting technique is used for solving the problem. or A Simultaneous approach is made. In this, we can solve problem either by formulating and solving an integer programming problem or by applying Langrangian relaxation based heuristic.
Table 3.9;6;5;4 : Input - Operation Sequence, Cost of Material Handling, Production Volume of Parts, Processing Time Required by Parts on Machines, Capacity of Machines			
Gupta et al [1996]	Genetic Algorithm	To minimize total movement and cell load variation	An integer programming problem is formulated and solved by genetic algorithm.
Verma and Ding [1995]	Heuristic	To minimize total material handling cost	In each iteration, each pair of cells considered for merging into a cell and change in total material flow cost due to this merging is calculated. Each machine is treated as a cell initially. Total material flow cost consist of intercell, intracell movement costs, backtracking cost and machine skipping cost. Pair of cell with most negative increase in total material handling cost is chosen for merging.
Table 3.9;8;3;1 : Input -Operation Sequence, Cost of Material Handling, Cost of Machines, Copy of Machines, Number of Cells			
Sarker and Yu [1994]	Heuristic followed by Linear Programming	To minimize bottlenecks	1. Computation of intercell flow matrix and distance matrix. 2. Assignment of cell along line is done by clustering technique in which initial assignment solution is computed and followed by generation of all possible solution. 3. Solve the formulated problem by linear programming approach.

Chapter 2

PROPOSED FORMULATION AND SOLUTION METHODOLOGY

This chapter deals with problem environment and its formulation. Also, a solution methodology is discussed to solve the resulting 0-1 integer programming formulation and in that a truncated tree search based heuristic algorithm is proposed. Firstly, problem environment is discussed followed by its formulation, solution methodology used and proposed algorithm. Complexity analysis of proposed algorithm is also presented. Finally, an idea about illustration of a numerical example is also given.

2.1 Problem Environment

In the present study, our aim is to solve a generalised grouping problem i.e. to form machine cells, using a non-binary part-machine input matrix. The problem environment is characterised by the following descriptors.

Parameters : The following factors related to the parts, machines, cell size and costs are pre-specified/known a priori.

Parts

- Processing time required by parts for each process
- Production volume of parts
- Operation sequence of parts
- Alternate process plans for each parts

Machines

- Number and types of machines

Cell Size

- Upper limit on cell size

Costs

- Costs related to intercell and intracell moves
- Cost of processing of parts.

Assumptions : In order to formulate the problem, the following assumptions are made :

1. A part may have more than one process plans.
2. Entire volume of part will go through one and only one process plan i.e. there is no batch splitting along the alternative process plans.
3. For a part, one and only one machine will be selected for an operation in a process plan.
4. There may be more than one copy of a machine type.
5. One machine will be assigned to one and only one machine cell i.e. one machine should not be shared by two or more than two cells.
6. A machine cell should not consist of more than one machine of the same type i.e. in case of multiple copies of a machine type one copy each will be assigned to different cells.

Objective : The objective is to form machine cells to minimize the sum of (i) processing costs, and (ii) material movement costs related with intercell and intracell movements. As the intercell and intracell movement costs would generally have different extents and impact on the efficiency of the cells, different weights are assigned to these movements to accommodate this fact. Further, some times the part movement (or material handling) may depend on the stage of the processing, i.e. a part towards final stages of processing may involve movement cost different than the one at initial stages. An alternate model is also suggested to capture this type of situation.

2.2 Problem Formulation

The problem is formulated as an integer programming model to minimize the sum of processing costs and material movement costs related with movements subject to some constraints.

Notation

Indices

p	:	part
r	:	process plan
o, o_1	:	operation
m, m_1	:	machine

i	:	machine type
c	:	cell

Parameters

N	:	Total number of parts
M	:	Total number of machines
M_i	:	Total number of machines of type i
I	:	Total number of machine types
R_p	:	Total number of process plans for part p
O_p^r	:	Total number of operations required by part p in rth process plan
V_p	:	Production volume of part p
$c_{p,m}^r$	:	Cost of processing part p per unit time using its rth process plan on machine m
$t_{p,m}^r$	:	Processing time per unit required by part p using its rth process plan on machine m
U^c	:	Upper limit on machine cell size
α	:	Material handling cost per unit related with intercell moves
β	:	Material handling cost per unit related with intracell moves
λ	:	Ratio of material handling cost and processing cost of the succeeding operation of a part related with intercell movement
μ	:	Ratio of material handling cost and processing cost of the succeeding of a part related with intracell movement

Indicator Variable

$$B_{m,i} = \begin{cases} 1 & \text{if machine } m \text{ is of type } i \\ 0 & \text{otherwise} \end{cases}$$

$$A_{p,m}^{r,o} = \begin{cases} 1 & \text{if part } p \text{ uses machine } m \text{ for its } o\text{th operation in } r\text{th process plan} \\ 0 & \text{otherwise} \end{cases}$$

If a part p uses machine m for its oth operation in rth process plan and there are more than one copy of machine m, then value of $A_{p,m}^{r,o}$ will be 1 for every copy of machine m.

Decision Variables

$$Z_p^r = \begin{cases} 1 & \text{if } r\text{th process plan is selected for part } p \\ 0 & \text{otherwise} \end{cases}$$

$$Y_m^c = \begin{cases} 1 & \text{if machine } m \text{ is assigned to cell } c \\ 0 & \text{otherwise} \end{cases}$$

$$X_{p,m}^{r,o} = \begin{cases} 1 & \text{if part } p \text{ uses machine } m \text{ for } o\text{th operation in } r\text{th process plan} \\ 0 & \text{otherwise} \end{cases}$$

If a part p uses machine m for its oth operation in rth process plan and there are more than one copy of machine m, then value of $X_{p,m}^{r,o}$ will be 1 for any one of the copy of machine m. For other machines of same type, value of $X_{p,m}^{r,o}$ will be zero.

If there is single copy of every machines available then $X_{p,m}^{r,o}$ will be equivalent to $A_{p,m}^{r,o}$.

2.2.1 Objective Function

As stated before, the objective is to minimize the sum of processing costs and material handling costs. Two models of objective function are presented in this section. First model deals with the problem environment where material handling cost is independent of the processing cost while the other model consider the condition where material handling cost is proportional to the processing cost.

(a) Model 1 : Independent Material Handling Cost and Processing Cost

The objective function of Model 1 is the sum of processing costs and material handling costs in which the material handling cost is independent of the processing cost and depends upon the production volume of the parts and the type of movement. The objective function has two terms. The first term is associated with the cost of processing incurred due to operations of a part which depends on its alternative process plans. The second term represents the material handling costs related to intercell and intracell movements of parts. Further, the second term also consist of two terms one is related with intercell movement and the other with intracell movement.

The cost of processing incurred due to oth operation of a part p is the product of production volume V_p of part p, processing time per unit required by and cost of processing per unit time incurred due to oth operation of that part in its rth process plan if rth process plan is used for part p. This cost is determined by obtaining the product of the production volume of part p, the cost of processing per unit time and the processing time

per unit required by part p using its rth process plan on a machine m, if machine m is required for its oth operation. Cost of processing for the oth operation of part p in its rth process plan is given as

$$= V_p \sum_{m=1}^{M} c_{p,m}^{r} t_{p,m}^{r} X_{p,m}^{r,o} . \tag{1}$$

Hence, for evaluating the total cost of processing of part p in its rth process plan it is required to sum equation (1) over the operations of part p in its rth process plan and it is given as

$$= V_p \sum_{o=1}^{O_p^r} \sum_{m=1}^{M} c_{p,m}^{r} t_{p,m}^{r} X_{p,m}^{r,o} . \tag{2}$$

Therefore, the total cost of processing can be shown as the summation of equation (2) over the parts and their process plans i.e.

$$= \sum_{p=1}^{N} V_p \sum_{r=1}^{R_p} Z_p^r \sum_{o=1}^{O_p^r} \sum_{m=1}^{M} c_{p,m}^{r} t_{p,m}^{r} X_{p,m}^{r,o} . \tag{3}$$

As stated before, the second term of the objective function is associated with the sum of material handling costs related to movements of materials. An intercell movement occurs for part p when machine m is required by it for o_1th operation in rth process plan where machine m belongs to cell c and some machine m_1 is required for its (o_1-1)th operation and machine m_1 does not belong to cell c. Here M, the total number of machines, is the maximum number of cells possible.

The intercell movements between two consecutive (o_1-1)th and o_1th operations for part p in rth process plan is as

$$= \sum_{c=1}^{M} Y_m^c \left[1 - \sum_{m_1=1}^{M} \left(X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c \right) \right] . \tag{4}$$

As mentioned while introducing the notation, α and β are the material movement cost per unit for intercell and intracell movements respectively. Then the material handling cost for o_1th operation related with this intercell move is

$$= V_p \alpha \sum_{c=1}^{M} Y_m^c \left[1 - \sum_{m_1=1}^{M} \left(X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c \right) \right] . \tag{5}$$

For (o_1-1)th and o_1th operations, if there is no intercell movement then there will

be an intracell movement. So, intracell movements between o_1th and (o_1-1)th operations for part p in its rth process plan is

$$= \sum_{c=1}^{M} Y_m^c \sum_{m_1=1}^{M} (X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c). \tag{6}$$

Thus, the material handling cost for o_1th operation related with this intracell movement will be

$$= V_p \beta \sum_{c=1}^{M} Y_m^c \sum_{m_1=1}^{M} (X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c). \tag{7}$$

The total material handling cost for o_1th operation of part p in its rth process plan

$$= \alpha V_p \sum_{c=1}^{M} Y_m^c \left[1 - \sum_{m_1=1}^{M} (X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c) \right] + \beta V_p \sum_{c=1}^{M} Y_m^c \sum_{m_1=1}^{M} (X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c). \tag{8}$$

Summing the equation (8) over the operations and machines required by part p in its rth process plan, we will get the total material handling cost for operations of part p in its rth process plan as

$$= V_p \sum_{o_1=2}^{O_p^r} \sum_{m=1}^{M} X_{p,m}^{r,o_1} \sum_{c=1}^{M} Y_m^c \left[\beta + (\alpha - \beta) \left\{ 1 - \sum_{m_1=1}^{M} \left(X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c \right) \right\} \right]. \tag{9}$$

For evaluating the total material handling cost for operations of all parts it is required to sum equation (9) over all the parts and their process plans, and we get the total material handling cost for operations for all parts as

$$= \sum_{p=1}^{N} V_p \sum_{r=1}^{R_p} Z_p^r \sum_{o_1=2}^{O_p^r} \sum_{m=1}^{M} X_{p,m}^{r,o_1} \sum_{c=1}^{M} Y_m^c \left[\beta + (\alpha - \beta) \left\{ 1 - \sum_{m_1=1}^{M} \left(X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c \right) \right\} \right]. \tag{10}$$

Our objective is to minimize the sum of processing costs and material handling costs, i.e. To minimize

$$\sum_{p=1}^{N} V_p \sum_{r=1}^{R_p} Z_p^r \sum_{o=1}^{O_p^r} \sum_{m=1}^{M} c_{p,m}^r t_{p,m}^r X_{p,m}^{r,o}$$

$$+ \sum_{p=1}^{N} V_p \sum_{r=1}^{R_p} Z_p^r \sum_{o_1=2}^{O_p^r} \sum_{m=1}^{M} X_{p,m}^{r,o_1} \sum_{c=1}^{M} Y_m^c \left[\beta + (\alpha - \beta) \left\{ 1 - \sum_{m_1=1}^{M} \left(X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c \right) \right\} \right]. \tag{11}$$

(b) Model 2 : Material Handling Cost Dependent on Processing Cost

Model 2 of the objective function is concerned with the sum of processing costs and material handling costs where material handling cost is dependent upon and the type of movement. (intercell or intracell) and also on the stage of processing and its cost of

processing. The movement cost is taken as proportion of the processing cost of the succeeding operation. The objective function has two terms. The first term as in Model 1, is associated with the cost of processing incurred due to operations of a part which depends on its alternative process plans. The second term represents the material handling costs related to intercell and intracell movements of parts.

As the cost of processing in this model is the same as that of Model 1, only the second term requires an explanation.

As stated before, the second term of the objective function is associated with the sum of material handling costs related to movements of materials. An intercell movement occurs for part p when machine m is required by it for o_1th operation in rth process plan where machine m belongs to cell c and some machine m_1 is required for its (o_1-1)th operation and machine m_1 does not belong to cell c. Here again, the maximum number of cells possible is M, the total number of machines.

The intercell movement between two consecutive (o_1-1)th and o_1th operations for part p in rth process plan is as

$$=\sum_{c=1}^{M} Y_m^c \left[1-\sum_{m_1=1}^{M} (X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c) \right]. \tag{12}$$

As mentioned while introducing the notation, λ and μ are the ratios of material movement cost and processing cost of a part operation for intercell and intracell movements respectively. It is assumed that λ and μ are constant for all parts and their operations. Then the material handling cost for o_1th operation related with this intercell move is

$$= V_p \lambda c_{p,m}^r t_{p,m}^r \sum_{c=1}^{M} Y_m^c \left[1-\sum_{m_1=1}^{M} (X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c) \right]. \tag{13}$$

Here again, for (o_1-1)th and o_1th operations, if there is no intercell movement then there will be an intracell movement. So, intracell movements between o_1th and (o_1-1)th operations for part p in its rth process plan is

$$= \sum_{c=1}^{M} Y_m^c \sum_{m_1=1}^{M} (X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c). \tag{14}$$

Thus, the material handling cost for o_1th operation related with this intracell movement will be

$$= V_p \mu c_{p,m}^r t_{p,m}^r \sum_{c=1}^{M} Y_m^c \sum_{m_1=1}^{M} (X_{p,m_1}^{r,(o_1-1)} Y_{m_1}^c). \tag{15}$$

The total material handling cost for o_1th operation of part p in its rth process plan

$$= \lambda V_p c^r_{p,m} t^r_{p,m} \sum_{c=1}^{M} Y^c_m \left[1 - \sum_{m_1=1}^{M} (X^{r,(o_1-1)}_{p,m_1} Y^c_{m_1}) \right] + \mu V_p c^r_{p,m} t^r_{p,m} \sum_{c=1}^{M} Y^c_m \sum_{m_1=1}^{M} (X^{r,(o_1-1)}_{p,m_1} Y^c_{m_1}). \quad (16)$$

Summing the equation (16) over the operations and machines required by part p in its rth process plan, we will get the total material handling cost for operations of part p in its rth process plan as

$$= V_p \sum_{o_1=2}^{O^r_p} \sum_{m=1}^{M} c^r_{p,m} t^r_{p,m} X^{r,o_1}_{p,m} \sum_{c=1}^{M} Y^c_m \left[\mu + (\lambda - \mu) \left\{ 1 - \sum_{m_1=1}^{M} \left(X^{r,(o_1-1)}_{p,m_1} Y^c_{m_1} \right) \right\} \right]. \quad (17)$$

For evaluating the total material handling cost for operations of all parts it is required to sum equation (17) over all the parts and their process plans, and we get the total material handling cost for operations for all parts as

$$= \sum_{p=1}^{N} V_p \sum_{r=1}^{R_p} Z^r_p \sum_{o_1=2}^{O^r_p} \sum_{m=1}^{M} c^r_{p,m} t^r_{p,m} X^{r,o_1}_{p,m} \sum_{c=1}^{M} Y^c_m \left[\mu + (\lambda - \mu) \left\{ 1 - \sum_{m_1=1}^{M} \left(X^{r,(o_1-1)}_{p,m_1} Y^c_{m_1} \right) \right\} \right]. \quad (18)$$

Thus from equations (3) and (18), the objective of minimising the sum of processing costs and material handling costs for Model 2 can be written as

To minimize

$$\sum_{p=1}^{N} V_p \sum_{r=1}^{R_p} Z^r_p \sum_{o=1}^{O^r_p} \sum_{m=1}^{M} c^r_{p,m} t^r_{p,m} X^{r,o}_{p,m}$$

$$+ \sum_{p=1}^{N} V_p \sum_{r=1}^{R_p} Z^r_p \sum_{o_1=2}^{O^r_p} \sum_{m=1}^{M} c^r_{p,m} t^r_{p,m} X^{r,o_1}_{p,m} \sum_{c=1}^{M} Y^c_m \left[\mu + (\lambda - \mu) \left\{ 1 - \sum_{m_1=1}^{M} \left(X^{r,(o_1-1)}_{p,m_1} Y^c_{m_1} \right) \right\} \right] \quad (19)$$

2.2.2 Constraints

(a) Assignment of process plan to a part

A part p is required to follow one and only one process plan r i.e.

$$\sum_{r=1}^{R_p} Z^r_p = 1 \qquad p = 1, ..., N. \quad (20)$$

(b) Assignment of machine to a cell

A machine m can be assigned to one and only one cell c. Here M, the total number of machines, will be the maximum number of cells possible also. Then the constraint reads as

$$\sum_{c=1}^{M} Y^c_m = 1 \qquad m = 1, ..., M. \quad (21)$$

(c) Limitation on cell size

A cell c should not contain more than U^c number of machines i.e.

$$\sum_{m=1}^{M} Y_m^c \leq U^c \qquad c = 1, \ldots, M. \qquad (22)$$

(d) Assignment of machine to a part for a specific operation

Only one machine m is to be selected for a part p for its oth operation in its rth process plan, i.e.

$$\sum_{m=1}^{M} X_{p,m}^{r,o} = 1 \qquad p = 1, \ldots, N; \qquad r = 1, \ldots, R_p;$$

$$o = 1, \ldots, O_p^r. \qquad (23)$$

(e) Cell machine type constraint

A cell should not contain more than one machine of same type i.e.

$$\sum_{m=1}^{M} B_{m,i} Y_m^c \leq 1 \qquad c = 1, \ldots, M; \qquad i = 1, \ldots, I. \qquad (24)$$

(f) Machines type availability constraint

Total number of machines assigned to cells of one type should be equal to the number of machines available of that type i.e.

$$\sum_{m=1}^{M} \sum_{c=1}^{M} B_{m,i} Y_m^c = M_i \qquad i = 1, \ldots, I. \qquad (25)$$

(g) Operation feasiblity constraint

A part cannot select a machine which is not required by any if its operation, i.e.

$$X_{p,m}^{r,o} \leq A_{p,m}^{r,o} \qquad p = 1, \ldots, N; \qquad r = 1, \ldots, R_p;$$

$$o = 1, \ldots, O_p^r; \qquad m = 1, \ldots, M. \qquad (26)$$

2.2.3 Indicator Variables

$$B_{m,i} \in \{0,1\} \qquad m = 1, \ldots, M; \qquad i = 1, \ldots, I.$$

$$A_{p,m}^{r,o} \in \{0,1\} \qquad p = 1, \ldots, N; \qquad r = 1, \ldots, R_p;$$

$$o = 1, \ldots, O_p^r \; ; \qquad m = 1, \ldots, M.$$

2.2.4 Decision Variables

$Z_p^r \in \{0,1\}$ $\quad p = 1, \ldots, N;$ $\quad r = 1, \ldots, R_p;$

$Y_m^c \in \{0,1\}$ $\quad m = 1, \ldots, M;$ $\quad c = 1, \ldots, M.$

$X_{p,m}^{r,o} \in \{0,1\}$ $\quad p = 1, \ldots, N;$ $\quad r = 1, \ldots, R_p;$

$\quad o = 1, \ldots, O_p^r;$ $\quad m = 1, \ldots, M.$

It is observed that this formulation has a large number of variables and constraints even for a small number of machines and parts. So, solving this integer programming problem by traditional mathematical programming technique would be very difficult. In order to overcome this, some heuristic approach would be a better alternative which may produce a good feasible solution not necessarily optimal.

2.3 Solution Methodology

The objective function and constraints of the problem are such that the problem is NP hard. Traditional mathematical programming approach is not appropriate for solving this problem because it will require excessive computation. Some non-conventional type of methods i.e. simulated annealing, genetic algorithm or tabu search techniques can be used to solve this problem and may yield good results but these techniques also require heavy computation. Other techniques like fuzzy clustering or neural network approach can solve this problem with lesser calculation but quality of solution may be poor. Truncated tree search heuristic can solve this problem and produce results comparable to optimal solution in less computation [Chang et al, 1996].

In truncated tree search heuristic, firstly a root node is created. This is known as basic node. After that, nodes are created by branching the root node. Branching can be carried out by any type of action depending on the problem environment. For our case, branching is done by assigning a machine to a cell from amongst the cells available. Such created nodes are called first level node. These nodes are checked for feasibility and all infeasible nodes are pruned. Now we define a number w which is essentially an integer and represents the maximum number of nodes to be branched further at one level. If number of feasible nodes at one level are less than or equal to w then all nodes are selected for branching purpose. If number of feasible nodes are greater than w then objective function for each node at that level is calculated and nodes are sorted according to value of their

objective function and out of all feasible nodes at that level, w nodes having better objective function are selected for further branching. In case of tie, nodes are chosen arbitrarily. Nodes which are not selected for further branching purpose, are pruned. This activity is called truncation of tree. Now the nodes selected for further branching are branched to form the nodes of next level and again feasiblity checking and truncation of tree are done. These procedures are repeated till branching stops. In this problem, branching is stopped when all machines are assigned or number of level is equal to total number of machines.

At last, out of the nodes of last level, node having the best objective function value is selected as final solution.

In the above solution methodology, quality of solution largely depends upon the truncating parameter w. If w is large then quality of solution will be better because for large value of w, a large number of nodes will be explored and tested before reaching the solution. On the other hand if w is small then there is possibility of getting poor results.

Computation time mainly depends upon the total number of machines and the truncating parameter w. Since this heuristic has polynomial complexity, therefore number of computations will increase with increase in the total number of machines. Previously discussed that if value of truncating parameter is large then certainly a large number of nodes will be explored and tested for getting final solution and requiring more computation time.

2.4 Proposed Algorithm for Cell Formation

Stage 1 : Initialisation

Place an initial node which represents that all machines are assigned to an artificial cell i.e. cell zero. List of nodes called OPEN contains only this zero level node.

Stage 2 : Branching Procedure

Branching is done from the node/nodes existing in list OPEN by removing one machine from cell zero and adding it to a machine cell amongst the cells available (referred to as real cell) one at a time and thereby generating all its successor nodes of the next level.

Stage 3 : Check for Feasibility

Feasiblity checking is done for all the successor nodes.

A node is called feasible if it satisfies both the following criteria

(i) a real cell is not containing more than a prespecified number of machines which is also referred as upper limit of cell size, and

(ii) a real cell is not containing more than one machine of same type.

All infeasible nodes are pruned.

Stage 4 : Updation of List of Nodes

If number of successor nodes at this level is less than or equal to truncating parameter w then an updation is done in list OPEN and node/nodes present in list OPEN is/are replaced by these successor nodes.

Stage 5 : Evaluation of Nodes and Updation of List of Nodes

If number of successor nodes at this level is greater than w then a function F is calculated.

For Model 1, F is given as

$$F = \sum_{p=1}^{N} V_p \min_{r=1}^{R_p} \left[\min_{m \in a(m)} \left\{ t^r_{p,m} c^r_{p,m} + \sum_{o=2}^{O^r_p} \min_{m \in d(m)} \left\{ t^r_{p,m} c^r_{p,m} + \left\{ \alpha b + \beta(1-b) \right\} \right\} \right\} \right]$$

Similarly for Model 2, F is given as

$$F = \sum_{p=1}^{N} V_p \min_{r=1}^{R_p} \left[\min_{m \in a(m)} \left\{ t^r_{p,m} c^r_{p,m} + \sum_{o=2}^{O^r_p} \min_{m \in d(m)} \left\{ t^r_{p,m} c^r_{p,m} \left\{ 1 + \lambda b + \mu(1-b) \right\} \right\} \right\} \right]$$

where

Sets $a(m)$ = { machines such that $A^{r,1}_{p,m} = 1$ }

$d(m)$ = { machines such that $A^{r,o}_{p,m} = 1$ } $\quad o = 1, \ldots, O^r_p$

$$b = \begin{cases} 1 & \text{if } o\text{th and } (o-1)\text{th operation for part } p \text{ in } r\text{th process plan are done in different cells} \\ 0 & \text{otherwise.} \end{cases}$$

Nodes having first w better values of F are kept and other nodes are pruned. In case of tie, nodes are chosen arbitrarily. Now an updation is made in list OPEN and existing node/nodes is/are replaced by these kept nodes.

Stage 6 : Stopping Criteria

If all machines are assigned to real cells then stop and select the node having the best value of F among the nodes of last level.

Else go to step 2.

2.5 Complexity of the Heuristic

To calculate the complexity of the heuristic in terms of M, N, R_p, L and w where

M is total number of machines

N is total number of parts

R_p is total number of number process plans for a part

L is the average number of copy of machines of one type

and w is the truncating parameter,

a step-by step analysis is carried out as follows.

Step 1 will be done once for any problem.

Considering the worst case, for assigning kth machine Step 2 will be repeated $(kw-w-1)$ times. The maximum number of cells to whom this machine can be assigned will be k for a node. If there are w nodes then minimum number of nodes could be (k-1), (k-2) or (k-w). So, maximum number of execution of step 2 will be $[k+(w-1)(k-1)]$.

Step 3 will be repeated $[k^2w-2kw+w+k]$ times. Because maximum number of cells in a node can be k and for others it could be $(k-1)$, $(k-2)$ or $(k-w)$. So maximum number of execution of step 3 will be $[(kw-w)(k-1)+k]$ considering that all nodes are feasible.

It is assumed that number of successor nodes are greater than w.

Step 4 will not be executed if the number of successor nodes are greater than w.

Step 5 will be executed $NR_p\ (kw-w+1)[L\ M\{L+(L^2-L)/2\}+(L^2-L)/2]+M(kw-w-1)(R_p^2-R_p)/2$ times. Because for an operation (except the first operation) of a part in one of its process plan, it is required to select a machine for which minimum sum of processing costs and material handling cost occur. For this, it is required to check whether there is an intercell or intracell movement. If there are L number of machines of each type, then for an operation of a part and for a machine, it is require to check whether there is an intercell or intracell movement and to evaluate the sum of processing cost and material handling costs for $[L+(L^2-L)/2]$ times. Considering the worst case where each part requires all the machines, it is required to repeat the related step $M[L+(L^2-L)/2]$ times for a machine. For selecting the machine for an operation, it is needed to evaluate the sum of processing costs and material handling costs incurred on the different machines required for an operation and to obtain the minimum one. For this, the related steps will be executed $[LM\{L-(L^2-$

$L)/2\}+(L^2-L)/2]$ times. For each process plan, this sum of processing costs and material handling costs will be evaluated and sorted. For this, related steps will be repeated $R_p[LM\{L+(L^2-L)/2\}+(L^2-L)/2]+(R_p^2-R_p)/2$ times. To sum it over the number of parts this procedure will be re-executed for $NR_p[LM\{L+(L^2-L)/2\}+(L^2-L)/2]+M(R_p^2-R_p)/2$ times. Since there are $(kw-w+1)$ nodes. So total number of repetition in step 5 will be $NR_p(kw-w+1)[LM\{L+(L^2-L)/2\}+(L^2-L)/2]+M(kw-w+1)(R_p^2-R_p)/2$

By adding all the repetitions required in these steps in the worst case, for k = 1 to M, the complexity of this heuristic comes out $O\left(M^3\{wL^3NR_p+w+wR_p^{\,2}\}\right)$.

2.6 An Illustration

The following problem is taken to illustrate the formulation and the solution methodology.

In this problem,

Total number of parts (N) = 10

Total number of process plans of part p (R_p) = 2 for each part

Production Volume of Part p (V_p) = 10 for each part

Range of processing time per unit required by part p using its rth process plan on machine m $(t^r_{p,m})$ and cost of processing of part p per unit time using its rth process plan on machine m $(c^r_{p,m})$ = 1 to 5.

Total number of machines (M) = 15

Total number of machine types (I) = 13

All machines are of different types except for machine type 1 and 2

There are two copies of machine type 1 and 2 i.e. machine 1 & 14 and machine 2 & 15 respectively.

Upper limit on cell size (U^c) = 5

Primary input data (processing time matrix, cost of processing matrix and operation sequence matrix) are tabulated in appendix A-1. Details of Model 1 objective function (equation 11), Model 2 objective function (equation 19) and constraints (equations 20 to 26) are presented in appendix A-2. Solution methodology used for solving the problem is illustrated in appendix A-3 and the final solution is presented in appendix A-4.

Chapter 3

RESULTS AND DISCUSSIONS

This chapter presents the experiences about the proposed formulation and the heuristic procedure to solve the problem. As stated in the previous chapter, the objective of the algorithm is to minimize sum of the processing costs and the material handling costs related with intercell and intracell movement. Several numerical problems encompassing a wide range of problem parameters are solved and the solutions are analysed to study the behavior of the grouping problem as well as the proposed solution methodology.

3.1 Problems and Their Solutions

For the proposed cell formation algorithm, a computer code is developed and a number of grouping problems have been solved using this computer code. The inputs for these problems consist of

- Number of parts
- Production volume of parts
- Operation sequence matrices representing the sequence of operations required by parts in each process plan of that part
- Processing time matrices representing the processing time required by that part in its each process plan on machines
- Cost processing matrices representing the cost of processing corresponding to processing time matrices
- Number of machines of each type
- Upper limit on the cell size or maximum number of machines that can be accommodated in one cell
- Truncating parameter
- Material handling cost per unit related with intercell and intracell movements in case of Model 1, or ratio of material handling costs related with intercell and intracell movement and processing cost of succeeding operation in case of problem environment which is related with Model 2.

Each input can have different levels. The major inputs are considered as follows.

(1) Size of the Problem : The problems have been grouped into two levels based on the size, viz. low and high. In the high level, there are thirty machines and twenty parts and in the low level, there are fifteen machines and ten parts.

(2) Routing Flexibility : For each level of problem size, there are three levels of routing flexibility viz. high, medium and low. In the high level of routing flexibility, there are three process plans for each part. In the medium level of routing flexibility, there are two process plan for each part and in the low level, there is one and only one process plan for each part.

(3) Processing Time and Cost of Processing : For each level of flexibility there are three levels of ranges each for the processing time and the cost of processing. In the low level, processing time and cost of processing for any part on any machine are kept same *i.e.* zero variance in processing time data and zero variance in cost of processing data. Cost of processing and processing time are the same for each part. In the medium level there is a small variance in processing time data and in the cost of processing data. In this case, the cost of processing and the processing time vary from one to five. In the high level, there is a large variance in processing time data and in the cost of processing data and is assumed to vary from two to ten.

(4) Production Volume : Furthermore, for each level of problem size, there are three levels of production volume range of parts. In low level there is zero variance in production volume data. Production volume of each part is ten. In the medium level, there is a low variance in production volume data. Production volume of parts may vary from ten to twenty four. In the high level, there is a large variance in production volume data. Production volume of parts vary from twenty to forty eight.

(5) Cell Size Limit : For high level of problem, maximum number of machines can be accommodate in one cell is taken as ten. Value of that for low level of problem is five.

(6) Truncating Parameter : The truncating parameter has five levels namely first, second, third, fourth and fifth. It is equal to one, two, three, four and five for first, second, third, fourth and fifth level respectively.

Therefore, two levels of problem size, three levels of flexibility, three levels of processing time and cost of processing ranges and three levels of production volume range, result into fifty four ($2\times3\times3\times3=54$) problems as a combination of these attributes.

The truncating parameter is kept constant equal to five for all these combination problems. Further, four problems are generated by varying the truncating parameter from its first

level to its fourth level in a high level problem with high routing flexibility, high level of processing time, cost of processing and production volume range.

For all the problems with Model 1 of the objective function, value of material handling cost per unit related with intercell and intracell movements *i.e.* α and β are set as 4 and 1 respectively.

Similarly for all the problems having Model 2 of the objective function, value of ratio of the material handling cost related with intercell and intracell movements, and the processing cost of succeeding operation *i.e.* λ and μ are taken as 1 and 0.2 respectively.

Therefore, all the fifty eight problems are solved twice once each for the two models - Model 1 and Model 2.

Table 3.1 gives the relevant values and the ranges of various problem parameters for all 58 problems chosen for experimentation.

In the following section, the above mentioned 58 problems and their solutions corresponding to both Model 1 and Model 2 are presented. These problems have been solved for deciding the machine groups and process plan for the parts. So in final solution, a machine-cell matrix representing the machines and the cell to which that machine is assigned, a part-process plan matrix representing the parts and the process plans selected for that part, user time required in solving the problem and value of the objective function which represents the corresponding total of processing costs and material handling costs are reported

Details of inputs for all fifty eight problems i.e. processing time matrices, operation sequence matrices and cost of processing matrices, are given in appendix A-5.

M	No. of machines	$t^r_{p,m}$	Processing cost
N	No. of parts	V_p	Production volume
R_p	No. of process plan for each part	w	Truncating parameter
$c^r_{p,m}$	Processing time	U^c	Upper limit on cell size

Table 3.1 : Example problems of different types according to levels of the parameters

Problem No.	M	N	R_p	Range of $c^r_{p,m}$	Range of $t^r_{p,m}$	Range of V_p	w	U^c
1	30	20	3	2 - 10	2 - 10	20 - 48	5	10
2	30	20	3	2 - 10	2 - 10	10 - 24	5	10
3	30	20	3	2 - 10	2 - 10	10 - 10	5	10
4	30	20	3	1 - 5	1 - 5	20 - 48	5	10
5	30	20	3	1 - 5	1 - 5	10 - 24	5	10
6	30	20	3	1 - 5	1 - 5	10 - 10	5	10
7	30	20	3	1 - 1	1 - 1	20 - 48	5	10
8	30	20	3	1 - 1	1 - 1	10 - 24	5	10
9	30	20	3	1 - 1	1 - 1	10 - 10	5	10
10	30	20	2	2 - 10	2 - 10	20 - 48	5	10
11	30	20	2	2 - 10	2 - 10	10 - 24	5	10
12	30	20	2	2 - 10	2 - 10	10 - 10	5	10
13	30	20	2	1 - 5	1 - 5	20 - 48	5	10
14	30	20	2	1 - 5	1 - 5	10 - 24	5	10
15	30	20	2	1 - 5	1 - 5	10 - 10	5	10
16	30	20	2	1 - 1	1 - 1	20 - 48	5	10
17	30	20	2	1 - 1	1 - 1	10 - 24	5	10
18	30	20	2	1 - 1	1 - 1	10 - 10	5	10
19	30	20	1	2 - 10	2 - 10	20 - 48	5	10
20	30	20	1	2 - 10	2 - 10	10 - 24	5	10
21	30	20	1	2 - 10	2 - 10	10 - 10	5	10
22	30	20	1	1 - 5	1 - 5	20 - 48	5	10
23	30	20	1	1 - 5	1 - 5	10 - 24	5	10
24	30	20	1	1 - 5	1 - 5	10 - 10	5	10
25	30	20	1	1 - 1	1 - 1	20 - 48	5	10
26	30	20	1	1 - 1	1 - 1	10 - 24	5	10
27	30	20	1	1 - 1	1 - 1	10 - 10	5	10
28	15	10	3	2 - 10	2 - 10	20 - 40	4	5
29	15	10	3	2 - 10	2 - 10	10 - 20	4	5
30	15	10	3	2 - 10	2 - 10	10 - 10	4	5
31	15	10	3	1 - 5	1 - 5	20 - 40	4	5
32	15	10	3	1 - 5	1 - 5	10 - 20	4	5
33	15	10	3	1 - 5	1 - 5	10 - 10	4	5
34	15	10	3	1 - 1	1 - 1	20 - 40	4	5
35	15	10	3	1 - 1	1 - 1	10 - 20	4	5
36	15	10	3	1 - 1	1 - 1	10 - 10	4	5
37	15	10	2	2 - 10	2 - 10	20 - 40	4	5
38	15	10	2	2 - 10	2 - 10	10 - 20	4	5

39	15	10	2	2 - 10	2 - 10	10 - 10	4	5
40	15	10	2	1 - 5	1 - 5	20 - 40	4	5
41	15	10	2	1 - 5	1 - 5	10 - 20	4	5
42	15	10	2	1 - 5	1 - 5	10 - 10	4	5
43	15	10	2	1 - 1	1 - 1	20 - 40	4	5
44	15	10	2	1 - 1	1 - 1	10 - 20	4	5
45	15	10	2	1 - 1	1 - 1	10 - 10	4	5
46	15	10	1	2 - 10	2 - 10	20 - 40	4	5
47	15	10	1	2 - 10	2 - 10	10 - 20	4	5
48	15	10	1	2 - 10	2 - 10	10 - 10	4	5
49	15	10	1	1 - 5	1 - 5	20 - 40	4	5
50	15	10	1	1 - 5	1 - 5	10 - 20	4	5
51	15	10	1	1 - 5	1 - 5	10 - 10	4	5
52	15	10	1	1 - 1	1 - 1	20 - 40	4	5
53	15	10	1	1 - 1	1 - 1	10 - 20	4	5
54	15	10	1	1 - 1	1 - 1	10 - 10	4	5
55	30	20	3	2 - 10	2 - 10	20 - 48	4	10
56	30	20	3	2 - 10	2 - 10	20 - 48	3	10
57	30	20	3	2 - 10	2 - 10	20 - 48	2	10
58	30	20	3	2 - 10	2 - 10	20 - 48	1	10

3.2 Discussions

It is observed from the results for the two objective functions in models 1 and 2 that there is a great impact of problem size, routing flexibility and truncating parameter on the solution, value of the objective functions and the performance of the proposed methodology.

Generally, CPU time is taken as measure of performance for any algorithm. Here user time in stead of CPU time is taken as performance measure of algorithm. This has been done so because in the present problem, CPU times are very small and any conclusion made on the basis of such small CPU times may not lead to any conclusive statement. User times are significantly large to draw a meaningful conclusion. Although user times are also dependent on CPU times and for just comparing the performance of algorithm for solving different problems in the same time, user time can be used as measure of performance.

3.2.1 Impact on User Time

(a) Impact of Level of the Problem Size

Problem Size	Problem No.	Average User Time
High (30 machines, 20 parts)	1 to 27	5.00 Sec.
Low (15 machines, 10 parts)	28 to 54	0.4 Sec.

This observation shows the effect of the level of problem size on user time that as expected the user time is higher for the large size problems as compared to small size problems.

(b) Impact of Level of the Routing Flexibility

Problem Size	Problem No.	No. of Process Plans for Each Part	Average user time
High (30 machines, 20 parts)	1 to 9	3	5.8 Sec.
High (30 machines, 20 parts)	10 to 18	2	4.9 Sec.
High (30 machines, 20 parts)	19 to 27	1	4.0 Sec.

This observation shows the effect of the level of routing flexibility on the user time. Again, for large problem size the user time, as expected, is higher for higher value of flexibility. For small size problems, reduction in user time with reduction in routing flexibility is also observed but it does not have any significant pattern.

(c) Impact of the Truncating Parameter

Problem Size	Problem No.	No. of Process Plans for Each Part	Ranges for Processing Cost and Processing Time	Range of Production Volume	Truncating Parameter	User Tine
High (30 machines, 20 parts)	1	3	2-10	20-48	5	5.41 Sec.
High (30 machines, 20 parts)	55	3	2-10	20-48	4	4.33 Sec.
High (30 machines, 20 parts)	56	3	2-10	20-48	3	3.41 Sec.
High (30 machines, 20 parts)	57	3	2-10	20-48	2	2.25 Sec.
High (30 machines, 20 parts)	58	3	2-10	20-48	1	1.21 Sec.

This observation shows the effect of the truncating parameter on the user time. The user time decreases with decrease in the value of truncating parameter. Similar observation is also made for small size problems but the relative magnitude of decrease in user time is less as compared to that in case of large size problems.

3.2.2 *Impact on Objective Function*

(a) Impact of the Truncating Parameter

By observing value of objective function in solution of problem number 1, 55, 56, 57 and 58, it is noticed that the reduction in the value of truncating parameter results in relatively worse solution. For small value of truncating parameter, this proposed algorithm for cell formation has to search for lesser number of nodes. Therefore, this results in worse solution than that for large value of truncating parameter.

(b) Impact of Level of the Routing Flexibility

It is observed that reduction in routing flexibility results in relatively worse solution. For high routing flexibility, a part has more paths for processing, so parts have more alternative paths for selecting one out of them as compared to lesser routing flexibility condition. Certainly there is always a probability to get better path for processing as compared to lesser routing flexibility condition. Therefore routing flexibility affects the objective function value.

3.2.3 *Impact on Solution*

Following observations have been recorded regarding impact of the level of different parameters on solutions :

(a) Impact of the Level of Production Volume Range

For large size of problems,

- Solution is not affected when the level of production volume range is decreased from high to medium for a particular level of routing flexibility and for a particular level of processing time and cost of processing range.
- When the level of processing time and cost of processing range is high and the level of routing flexibility is high and low, then there is no effect of the level production volume range.
- When the level of processing time and cost of processing range is medium and the level of routing flexibility is high, then there is no effect of the level production volume range in case of Model 2.
- When the level of routing flexibility is low, then there is no effect of the level production volume range irrespective of other parameters in case of Model 1.

For small size of problems,

- Solution is not affected by the level of production volume range for high level of routing flexibility irrespective of other parameters viz. processing time and cost of processing range.
- When the level of routing flexibility is medium and the level of processing time and cost of processing range is medium and high then there is no effect of the level production volume range on solution.
- When the level of routing flexibility is low then there is no effect of the level production volume range on solution irrespective of the level of processing time and cost of processing range in case of Model 2.
- When the level of routing flexibility is low and the level of processing time and cost of processing range is medium and low then there is no effect of the level production volume range on solution in case of Model 1.

(b) Impact of Level of the Processing Time and Cost of Processing Range

For large size problems,

- There is no effect of the level of processing time and cost of processing range for a particular level of production volume range when they are decreased from medium to low and the level of routing flexibility is high or low in case of Model 1. Effect, however, is observed for the case of medium value of routing flexibility.
- There is no effect of the level of processing time and cost of processing range for medium and high level of production volume range when they are decreased from medium to low and the level of routing flexibility is high in case of Model 2.
- There is no effect of the level of processing time and cost of processing range for medium and high level of production volume range when they are decreased from high to medium and the level of routing flexibility is medium.

For small size problems,

- There is no effect of the level of processing time and cost of processing range when the level of routing flexibility is high.
- There is no effect of the level of processing time and cost of processing range when they are decreased from high to medium and the level of routing flexibility is medium.
- There is no effect of the level of processing time and cost of processing range when they are decreased from high to medium and the level of routing flexibility is low in case of Model 2.

- There is no effect of the level of processing time and cost of processing range when they are decreased from medium to low and the level of routing flexibility is low in case of Model 1.

(c) Impact of Level of the Routing Flexibility

This affects the solution in every case.

(d) Impact of Level of the Problem Size

For small size problem, it is observed that there is lesser effect of the levels of these parameters (*i.e.* processing time and cost of processing range, routing flexibility, production volume range) as compared to large size problem.

For this case, lesser number of nodes are formed at the levels of solution procedure and there is more chance of getting the same nodes for different problems as compared to the case of large problem size. Therefore, for small size problem, the number of different solution is less and this results in observation that there is lesser effect of the levels of these attributes on solution.

Any concrete conclusion cannot be made on the basis of the observations related with impact of the level of production volume range, the level of the processing time and cost of processing range and the level of routing flexibility because data are not sufficient. For drawing any meaningful conclusion, it is required to have more experimentation for a large range of parameters.

Problem Number 1

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 21, 23
3	12, 13, 14, 15, 16, 18, 19, 20
4	17, 22, 24, 25, 26, 27, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 21, 23
3	12, 13, 14, 15, 16, 18, 19, 20
4	17, 22, 24, 25, 26, 27, 28, 29, 30

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or 3	1	1,2 or 3	1 or 3	2	2	3	1	3	1 or 3	2
Process Plan (Model 2)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or 3	1	1,2 or 3	1 or 3	2	2	3	1	3	1 or 3	2

User Time (Model 1) : 5.41 Sec. **User Time** (Model 2) : 5.41 Sec.

Objective Function Value (Model 1) : 133802 **Objective Function Value** (Model 2) : 148263

Problem Number 2

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 21, 23
3	12, 13, 14, 15, 16, 18, 19, 20
4	17, 22, 24, 25, 26, 27, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 21, 23
3	12, 13, 14, 15, 16, 18, 19, 20
4	17, 22, 24, 25, 26, 27, 28, 29, 30

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or 3	1	1,2 or 3	1 or 3	2	2	3	1	3	1 or 3	2
Process Plan (Model 2)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or 3	1	1,2 or 3	1 or 3	2	2	3	1	3	1 or 3	2

User Time (Model 1) : 5.41 Sec. **User Time** (Model 2) : 5.41 Sec.

Objective Function Value (Model 1) : 66901 **Objective Function Value** (Model 2) : 74123

Problem Number 3

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 21, 23
3	12, 13, 14, 15, 16, 18, 19, 20
4	17, 22, 24, 25, 26, 27, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 21, 23
3	12, 13, 14, 15, 16, 18, 19, 20
4	17, 22, 24, 25, 26, 27, 28, 29, 30

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or 3	1	1,2 or 3	1 or 3	2	2	3	1	3	1 or 3	2
Process Plan (Model 2)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or 3	1	1,2 or 3	1 or 3	2	2	3	1	3	1 or 3	2

User Time (Model 1) : 6.12 Sec. **User Time** (Model 2) : 6.12 Sec.

Objective Function Value (Model 1) : 38940 **Objective Function Value** (Model 2) : 43132

Problem Number 4

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 22, 23, 24, 25, 26, 27, 28, 29, 30
3	12, 13
4	14, 15, 16, 17, 18, 19, 20, 21

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 22, 23, 24, 25, 26, 27, 28, 29, 30
3	12, 13
4	14, 15, 16, 17, 18, 19, 20, 21

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or 3	1	1,2 or 3	1 or 3	2	2	3	1	3	1 or 3	2
Process Plan (Model 2)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 o 3	1	1,2 or 3	1 or 3	2	2	3	1	3	1 o 3	2

User Time (Model 1) : 5.64 Sec. **User Time** (Model 2) : 5.64 Sec.

Objective Function Value (Model 1) : 34432 **Objective Function Value** (Model 2) :36780

Problem Number 5

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 22, 23, 24, 25, 26, 27, 28, 29, 30
3	12, 13
4	14, 15, 16, 17, 18, 19, 20. 21

Model 2

Cell No.	Machines
1	1, 2, 4, 5, 6, 7, 8, 9, 10, 11
2	3, 22, 23, 24, 25, 26, 27, 28, 29, 30
3	12, 13
4	14, 15, 16, 17, 18, 19, 20. 21

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 o 3	3	1 o 3	1	1,2 or 3	1 o 3	2	2	3	1	3	1 o 3	2
Process Plan (Model 2)	3	2	3	2 o 3	3	1 o 3	1,2 or 3	2 o 3	3	1 o 2	1	1,2 or 3	1 o 3	2	2	3	1	3	1 o 3	2

User Time (Model 1) : 5.65 Sec. **User Time** (Model 2) : 5.65 Sec.

Objective Function Value (Model 1) : 17476 **Objective Function Value** (Model 2) :18658

Problem Number 6

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1. 2. 3. 4, 5, 7, 8, 9. 10, 11
2	6. 17.
3	12. 13, 14, 15, 16, 18, 19. 20, 21
4	22. 23. 24, 25, 26, 27, 28. 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8. 9, 10, 11
2	6, 22. 23, 24, 25. 26, 27, 28. 29, 30
3	12, 13
4	14, 15, 16, 17. 18. 19, 20. 21

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 o 3	3	1 o 3	1	1,2 or 3	1 o 3	2	2	3	1	3	1 o 3	2
Process Plan (Model 2)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 o 3	3	1 o 3	1	1,2 or 3	1 o 3	2	2	3	1	3	1 o 3	2

User Time (Model 1) : 6.22 Sec. **User Time** (Model 2) : 6.22 Sec.

Objective Function Value (Model 1) : 10170 **Objective Function Value** (Model 2) :10864

Problem Number 7

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 22, 23, 24, 25, 26, 27, 28, 29, 30
3	12, 13
4	14, 15, 16, 17, 18, 19, 20, 21

Model 2

Cell No.	Machines
1	1, 2, 4, 5, 6, 7, 8, 9, 10, 11
2	3, 22, 23, 24, 25, 26, 27, 28, 29, 30
3	12, 13
4	14, 15, 16, 17, 18, 19, 20, 21

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 o 3	3	1 o 3	1	1,2 or 3	1 o 3	2	2	3	1	3	1 o 3	2
Process Plan (Model 2)	3	2	3	2 o 3	3	1 o 3	1,2 or 3	2 o 3	3	1 o 2	1	1,2 or 3	1 o 3	2	2	3	1	3	1 o 3	2

User Time (Model 1) : 5.59 Sec. **User Time** (Model 2) : 5.59 Sec.

Objective Function Value (Model 1) : 3662 **Objective Function Value** (Model 2) :2317

Problem Number 8

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 22, 23, 24, 25, 26, 27, 28, 29, 30
3	12, 13
4	14, 15, 16, 17, 18, 19, 20, 21

Model 2

Cell No.	Machines
1	1, 2, 4, 5, 6, 7, 8, 9, 10, 11
2	3, 22, 23, 24, 25, 26, 27, 28, 29, 30
3	12, 13
4	14, 15, 16, 17, 18, 19, 20, 21

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 o 3	3	1 o 3	1	1,2 or 3	1 o 3	2	2	3	1	3	1 o 3	2
Process Plan (Model 2)	3	2	3	2 o 3	3	1 o 3	1,2 or 3	2 o 3	3	1 o 2	1	1,2 or 3	1 o 3	2	2	3	1	3	1 o 3	2

User Time (Model 1) : 5.46 Sec. **User Time** (Model 2) : 5.46 Sec.

Objective Function Value (Model 1) : 1831 **Objective Function Value** (Model 2) :1151

Problem Number 9

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 17
3	12, 13, 14, 15, 16, 18, 19, 20, 21, 23
4	22, 24, 25, 26, 27, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 8, 9, 10, 11
2	6, 17
3	12, 13, 14, 15, 16, 18, 19, 20, 21, 23
4	22, 24, 25, 26, 27, 28, 29, 30

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or	1	1,2 or 3	1 or 3	2 or 3	2 or 3	3	1	3	1 or 3	2
Process Plan (Model 2)	3	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3.	3	1 or 3	1	1,2 or 3	1 o 3	2 o 3	2 o 3	3	1	3	1 o 3	2

User Time (Model 1) : 4.94 Sec. **User Time** (Model 2) : 4.94 Sec.

Objective Function Value (Model 1) : 1060 **Objective Function Value** (Model 2) :676

Problem Number 10

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 14
2	5, 7, 12
3	13, 15, 16, 17, 18, 19, 20, 22, 23, 27
4	21, 24, 25, 26, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 21, 27, 30
3	13, 14, 18, 19, 20, 22, 23, 25, 26, 29
4	15, 16, 17, 24, 28

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	2	1 or 2	1	2	1 or 2	2	2	1	1	1 or 2	1	2	2	2	1	2	1 o 2	2
Process Plan (Model 2)	2	2	1 o 2	1 o 2	2	2	1 o 2	2	2	1	1 o 2	1 o 2	1	2	2	2	2	2	1	2

User Time (Model 1) : 5.05 Sec. **User Time** (Model 2) : 5.05 Sec.

Objective Function Value (Model 1) : 135654 **Objective Function Value** (Model 2) :154056

Problem Number 11

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 14
2	5, 7, 12
3	13, 15, 16, 17, 18, 19, 20, 22, 23, 27
4	21, 24, 25, 26, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 21, 27, 30
3	13, 14, 18, 19, 20, 22, 23, 25, 26, 29
4	15, 16, 17, 24, 28

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	2	1 or 2	1	2	1 or 2	2	2	1	1	1 or 2	1	2	2	2	1	2	1 o 2	2
Process Plan (Model 2)	2	2	1 o 2	1 o 2	2	2	1 o 2	2	2	1	1 o 2	1 o 2	1	2	2	2	2	2	1	2

User Time (Model 1) : 5.00 Sec. **User Time** (Model 2) : 5.00 Sec.

Objective Function Value (Model 1) : 67827 **Objective Function Value** (Model 2) :77020

Problem Number 12

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 13
2	5, 7, 14, 21, 27, 30
3	12, 17, 22, 23
4	15, 16, 18, 19, 20, 24, 25, 26, 28, 29

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 13
2	5, 7, 12, 14, 16, 21, 23, 27, 30
3	15, 18, 19, 20, 24, 25, 26, 29
4	17, 22, 28

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	1 or 2	1 or 2	2	2	1 o 2	2	2	1	1 or 2	1 or 2	1	2	2	2	2	2	2	2
Process Plan (Model 2)	2	2	1 o 2	1 o 2	2	2	1 o 2	2	2	1	1	1 o 2	1	2	2	2	2	1	1	2

User Time (Model 1) : 5.12 Sec. **User Time** (Model 2) : 5.12 Sec.

Objective Function Value (Model 1) : 39400**Objective Function Value** (Model 2) :45396

Problem Number 13

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 14
2	5, 7, 12
3	13, 15, 16, 17, 18, 19, 20, 22, 23, 27
4	21, 24, 25, 26, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 21, 27, 30
3	13, 14, 22, 23, 24, 25, 26, 29
4	15, 16, 17, 18, 19, 20, 28

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	2	1 or 2	1	2	1 or 2	2	2	1	1	1 o 2	1	2	2	2	1	2	1 o 2	2
Process Plan (Model 2)	2	2	1 or 2	1 or 2	2	2	1 or 2	2	2	1	1 or 2	1 or 2	1	2	2	2	2	2	2	2

User Time (Model 1) : 5.09 Sec. **User Time** (Model 2) : 5.09 Sec.

Objective Function Value (Model 1) : 35452 **Objective Function Value** (Model 2) :38322

Problem Number 14

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 14
2	5, 7, 12
3	13, 15, 16, 17, 18, 19, 20, 22, 23, 27
4	21, 24, 25, 26, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 21, 27, 30
3	13, 14, 22, 23, 24, 25, 26, 29
4	15, 16, 17, 18, 19, 20, 28

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	2	1 or 2	1	2	1 or 2	2	2	1	1	1 o 2	1	2	2	2	1	2	1 o 2	2
Process Plan (Model 2)	2	2	1 or 2	1 or 2	2	2	1 or 2	2	2	1	1 or 2	1 or 2	1	2	2	2	2	2	2	2

User Time (Model 1) : 5.03 Sec. **User Time** (Model 2) : 5.03 Sec.

Objective Function Value (Model 1) : 17726 **Objective Function Value** (Model 2) :19154

Problem Number 15

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 13
2	5, 7, 14
3	12, 15, 16, 17, 18, 19, 20, 21, 22, 23
4	24, 25, 26, 27, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 13
2	5, 7, 12, 14, 16, 21, 23, 27, 29, 30
3	15, 18, 19, 20, 24, 25, 26
4	17, 22, 28

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	1 or 2	1 or 2	1	2	1 or 2	2	2	1	1	1 or 2	1	2	2	2	1	2	1 or 2	2
Process Plan (Model 2)	2	2	1 or 2	1 or 2	1	2	1 or 2	2	2	1	1	1 or 2	1	2	2	2	1	2	1	2

User Time (Model 1) : 5.03 Sec. **User Time** (Model 2) : 5.03 Sec.

Objective Function Value (Model 1) : 10290 **Objective Function Value** (Model 2) :11174

Problem Number 16

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 13, 21, 23,
3	14, 15, 16, 18, 19, 20, 24, 25, 26, 27
4	17, 22, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 13, 21, 23
3	14, 15, 16, 18, 19, 20, 24, 25, 26, 27
4	17, 22, 28, 29, 30

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	1 or 2	1 or 2	2	2	1 or 2	2	2	1	1 or 2	1 or 2	1 or 2	2	2	2	1 or 2	1	2	2
Process Plan (Model 2)	2	2	1 or 2	1 or 2	2	2	1 or 2	2	2	1	1 or 2	1 or 2	1 or 2	2	2	2	1 or 2	1	2	2

User Time (Model 1) : 5.02 Sec. **User Time** (Model 2) : 5.02 Sec.

Objective Function Value (Model 1) : 4774 **Objective Function Value** (Model 2) :2722

Problem Number 17

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 13, 21, 23,
3	14, 15, 16, 18, 19, 20, 24, 25, 26, 27
4	17, 22, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 13, 21, 23
3	14, 15, 16, 18, 19, 20, 24, 25, 26, 27
4	17, 22, 28, 29, 30

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	1 or 2	1 or 2	2	2	1 or 2	2	2	1	1 or 2	1 or 2	1 or 2	2	2	2	1 or 2	1	2	2
Process Plan (Model 2)	2	2	1 or 2	1 or 2	2	2	1 or 2	2	2	1	1 or 2	1 or 2	1 or 2	2	2	2	1 or 2	1	2	2

User Time (Model 1) : 5.04 Sec. **User Time** (Model 2) : 5.04 Sec.

Objective Function Value (Model 1) : 2387 **Objective Function Value** (Model 2) :1355

Problem Number 18

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 13, 7, 21, 22, 23,
3	14, 15, 16, 17, 18, 19, 20
4	24, 25, 26, 27, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 6, 8, 9, 10, 11, 12
2	5, 7, 13, 14, 21, 22, 23
3	15, 17, 18, 19, 20, 24, 25, 26, 28, 30
4	16, 27, 29

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	2	2	1 o 2	1 or 2	2	2	1 or 2	2	2	1	1 or 2	1 or 2	1 or 2	2	2	2	1	1	2	1
Process Plan (Model 2)	2	2	1 or 2	1 or 2	1	2	1 or 2	2	2	1	1 or 2	1 or 2	1	2	2	2	1	1	2	2

User Time (Model 1) : 4.26 Sec. **User Time** (Model 2) : 4.26 Sec.

Objective Function Value (Model 1) : 1340 **Objective Function Value** (Model 2) :784

Problem Number 19

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 18, 20, 26, 27, 28, 30
3	6, 7, 15, 16, 17, 19, 21
4	22, 23, 24, 25, 29

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 8, 9, 10, 11, 13
2	6, 7, 12, 16, 17, 19, 21, 24, 28, 30
3	14, 15, 18, 20, 27
4	22, 23, 25, 26, 29

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 4.09 Sec. **User Time** (Model 2) : 4.09 Sec.

Objective Function Value (Model 1) : 138988 **Objective Function Value** (Model 2) :165010

Problem Number 20

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 18, 20, 26, 27, 28, 30
3	6, 7, 15, 16, 17, 19, 21
4	22, 23, 24, 25, 29

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 8, 9, 10, 11, 13
2	6, 7, 12, 16, 17, 19, 21, 24, 28, 30
3	14, 15, 18, 20, 27
4	22, 23, 25, 26, 29

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 4.08 Sec. **User Time** (Model 2) : 4.08 Sec.

Objective Function Value (Model 1) : 69494 **Objective Function Value** (Model 2) :82498

Problem Number 21

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 18, 20, 26, 27, 28, 30
3	6, 7, 15, 16, 17, 19, 21
4	22, 23, 24, 25, 29

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 8, 9, 10, 11, 13
2	6, 7, 12, 16, 17, 19, 21, 24, 28, 30
3	14, 15, 18, 20, 27
4	22, 23, 25, 26, 29

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 3.88 Sec. **User Time** (Model 2) : 3.88 Sec.

Objective Function Value (Model 1) : 48120 **Objective Function Value** (Model 2) :46848

Problem Number 22

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 15, 18, 22, 25, 29
3	6, 7, 16, 17, 19, 21, 26, 27, 28, 30
4	20, 23, 24

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 8, 9, 10, 11, 13
2	6, 7, 12, 16, 17, 19, 21, 22, 25, 29
3	14, 15, 18, 26, 27, 28, 30
4	20, 23, 24

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 4.16 Sec. **User Time** (Model 2) : 4.16 Sec.

Objective Function Value (Model 1) : 36142 **Objective Function Value** (Model 2) :39833

Problem Number 23

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 15, 18, 22, 25, 29
3	6, 7, 16, 17, 19, 21, 26, 27, 28, 30
4	20, 23, 24

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 8, 9, 10, 11, 13
2	6, 7, 12, 16, 17, 19, 21, 22, 25, 29
3	14, 15, 18, 26, 27, 28, 30
4	20, 23, 24

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 4.08 Sec. **User Time** (Model 2) : 4.08 Sec.

Objective Function Value (Model 1) : 18331 **Objective Function Value** (Model 2) :20186

Problem Number 24

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 15, 18, 22, 25, 29
3	6, 7, 16, 17, 19, 21, 26, 27, 28, 30
4	20, 23, 24

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 8, 9, 10, 11, 14
2	6, 7, 12, 15, 16, 17, 19, 21, 22, 25
3	13, 18, 26, 27, 28, 30
4	20, 23, 24, 29

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 4.04 Sec. **User Time** (Model 2) : 4.04 Sec.

Objective Function Value (Model 1) : 10670 **Objective Function Value** (Model 2) :11650

Problem Number 25

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 15, 18, 22, 25, 29
3	6, 7, 16, 17, 19, 21, 26, 27, 28, 30
4	20, 23, 24

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 8, 9, 10, 11, 13
2	5, 6, 7, 15, 16, 17, 19, 21, 22
3	14, 18, 25, 26, 27, 28, 30
4	20, 23, 24, 29

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 4.17 Sec. **User Time** (Model 2) : 4.17 Sec.

Objective Function Value (Model 1) : 6112 **Objective Function Value** (Model 2) :3632

Problem Number 26

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Medium level	(10 - 24)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 15, 18, 22, 25, 29
3	6, 7, 16, 17, 19, 21, 26, 27, 28, 30
4	20, 23, 24

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 8, 9, 10, 11, 13
2	5, 6, 7, 15, 16, 17, 19, 21, 22
3	14, 18, 25, 26, 27, 28, 30
4	20, 23, 24, 29

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 4.18 Sec. **User Time** (Model 2) : 4.18 Sec.

Objective Function Value (Model 1) : 3056 **Objective Function Value** (Model 2) :1810

Problem Number 27

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 15, 18, 22, 25, 29
3	6, 7, 16, 17, 19, 21, 26, 27, 28, 30
4	20, 23, 24

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 8, 9, 10, 11, 12, 13
2	5, 14, 15, 18, 22, 25
3	6, 7, 16, 17, 19, 21, 26, 28, 30
4	20, 23, 24, 27, 29

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 4.10 Sec. **User Time** (Model 2) : 4.10 Sec.

Objective Function Value (Model 1) : 1860 **Objective Function Value**(Model 2) :1094

Problem Number 28

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.41 Sec. **User Time** (Model 2) : 0.41 Sec.

Objective Function Value (Model 1) : 56218 **Objective Function Value**(Model 2) :66643

Problem Number 29

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.41 Sec. **User Time** (Model 2) : 0.41 Sec.

Objective Function Value (Model 1) : 28139 **Objective Function Value**(Model 2) 33319

Problem Number 30

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.40 Sec. **User Time** (Model 2) : 0.40 Sec.

Objective Function Value (Model 1) : 18130 **Objective Function Value**(Model 2) :21520

Problem Number 31

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.38 Sec. **User Time** (Model 2) : 0.38 Sec.

Objective Function Value (Model 1) : 14716 **Objective Function Value**(Model 2) :15789

Problem Number 32

Levels of parameters :

Problem size	High level	(15 machines; 10 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.40 Sec. **User Time** (Model 2) : 0.40 Sec.

Objective Function Value (Model 1) : 7358 **Objective Function Value**(Model 2) :7892

Problem Number 33

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.40 Sec. **User Time** (Model 2) : 0.40 Sec.

Objective Function Value (Model 1) : 4750 **Objective Function Value**(Model 2) :5088

Problem Number 34

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.45 Sec. **User Time** (Model 2) : 0.45 Sec.

Objective Function Value (Model 1) : 1604 **Objective Function Value**(Model 2) :940

Problem Number 35

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.45 Sec. **User Time** (Model 2) : 0.45 Sec.

Objective Function Value (Model 1) : 802 **Objective Function Value**(Model 2) :468

Problem Number 36

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts).
Routing flexibility	High level	(3 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 7
2	5, 6, 8
3	9, 12, 13, 14
4	10, 11, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2
Process Plan (Model 2)	1 or 2	2	1 or 3	1 or 2	1 or 3	2	2	1	3	2

User Time (Model 1) : 0.45 Sec. **User Time** (Model 2) : 0.45 Sec.

Objective Function Value (Model 1) : 530 **Objective Function Value**(Model 2) :308

Problem Number 37

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.38 Sec. **User Time** (Model 2) : 0.38 Sec.

Objective Function Value (Model 1) : 57120 **Objective Function Value**(Model 2) :63172

Problem Number 38

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.40 Sec. **User Time** (Model 2) : 0.40 Sec.

Objective Function Value (Model 1) : 28560 **Objective Function Value**(Model 2) :31583

Problem Number 39

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.37 Sec. **User Time** (Model 2) : 0.37 Sec.

Objective Function Value (Model 1) : 18380 **Objective Function Value**(Model 2) :20352

Problem Number 40

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.37 Sec. **User Time** (Model 2) : 0.37 Sec.

Objective Function Value (Model 1) : 14694 **Objective Function Value**(Model 2) :16658

Problem Number 41

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.39 Sec. **User Time** (Model 2) : 0.39 Sec.

Objective Function Value (Model 1) : 7347 **Objective Function Value**(Model 2) :8328

Problem Number 42

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.35 Sec. **User Time** (Model 2) : 0.35 Sec.

Objective Function Value (Model 1) : 4730 **Objective Function Value**(Model 2) :5360

Problem Number 43

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.44 Sec. **User Time** (Model 2) : 0.44 Sec.

Objective Function Value (Model 1) : 1326 **Objective Function Value**(Model 2) :885

Problem Number 44

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Low levei	(1 - 1; 1 - 1)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 11
3	9
4	12, 13, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4. 10
2	5. 6, 7, 8, 11
3	9
4	12, 13. 14. 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.41 Sec. **User Time** (Model 2) : 0.41 Sec.

Objective Function Value (Model 1) : 663 **Objective Function Value**(Model 2) :440

Problem Number 45

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Medium level	(2 process plans for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 13
3	9, 11
4	12, 14, 15

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 10
2	5, 6, 7, 8, 13
3	9, 11
4	12, 14, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	2	1	1 or 2	2	2	2	1	1	2
Process Plan (Model 2)	1	2	1	1 or 2	2	2	2	1	1	2

User Time (Model 1) : 0.33 Sec. **User Time** (Model 2) : 0.33 Sec.

Objective Function Value (Model 1) : 430 **Objective Function Value**(Model 2) :288

Problem Number 46

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 9
2	5, 11, 12
3	6, 7, 8, 10, 14
4	13, 15

Model 2

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.34 Sec. **User Time** (Model 2) : 0.34 Sec.

Objective Function Value (Model 1) : 57570 **Objective Function Value**(Model 2) :66841

Problem Number 47

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 9
2	5, 11, 12
3	6, 7, 8, 10, 14
4	13, 15

Model 2

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.33 Sec. **User Time** (Model 2) : 0.33 Sec.

Objective Function Value (Model 1) : 28785 **Objective Function Value**(Model 2) :33419

Problem Number 48

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Model 2

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.36 Sec. **User Time** (Model 2) : 0.36 Sec.

Objective Function Value (Model 1) : 18530 **Objective Function Value**(Model 2) :21440

Problem Number 49

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Model 2

Cell No.	Machines
1	1, 3. 5, 10, 11
2	2, 4. 9, 12
3	6. 7, 8, 14
4	13. 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.36Sec. **User Time** (Model 2) : 0.36Sec.

Objective Function Value (Model 1) : 15144 **Objective Function Value**(Model 2) :16708

Problem Number 50

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1. 3. 5. 10. 11
2	2, 4, 9, 12
3	6, 7, 8. 14
4	13, 15

Model 2

Cell No.	Machines
1	1. 3. 5. 10. 11
2	2. 4. 9. 12
3	6, 7. 8. 14
4	13. 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.34Sec. **User Time** (Model 2) : 0.34Sec.

Objective Function Value (Model 1) : 7572 **Objective Function Value**(Model 2) :8352

Problem Number 51

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of processing costs and costs of processing	Medium level	(1 - 5; 1 - 5)
Range of production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Model 2

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.33Sec. **User Time** (Model 2) : 0.33Sec.

Objective Function Value (Model 1) : 4880 **Objective Function Value**(Model 2) :5380

Problem Number 52

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	High level	(20 - 40)

Solutions : Model 1

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Model 2

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.31Sec. **User Time** (Model 2) : 0.31Sec.

Objective Function Value (Model 1) : 1776 **Objective Function Value**(Model 2) :1007

Problem Number 53

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Medium level	(10 - 20)

Solutions : Model 1

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Model 2

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.31Sec. **User Time** (Model 2) : 0.31Sec.

Objective Function Value (Model 1) : 888 **Objective Function Value**(Model 2) :502

Problem Number 54

Levels of parameters :

Problem size	Low level	(15 machines; 10 parts)
Routing flexibility	Low level	(One process plan for each part)
Range of Processing costs and costs of processing	Low level	(1 - 1; 1 - 1)
Range of Production volume of parts	Low level	(10 - 10)

Solutions : Model 1

Cell No.	Machines
1	1, 3, 5, 10, 11
2	2, 4, 9, 12
3	6, 7, 8, 14
4	13, 15

Model 2

Cell No.	Machines
1	1, 3, 5, 8, 10
2	2, 4, 9, 11, 12
3	6, 7, 14
4	13, 15

Parts	1	2	3	4	5	6	7	8	9	10
Process Plan (Model 1)	1	1	1	1	1	1	1	1	1	1
Process Plan (Model 2)	1	1	1	1	1	1	1	1	1	1

User Time (Model 1) : 0.33Sec. **User Time** (Model 2) : 0.33Sec.

Objective Function Value (Model 1) : 580 **Objective Function Value**(Model 2) :328

Problem Number 55

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for every part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 9, 10, 13, 15
2	6, 8, 11, 12, 16, 18, 19, 20, 21, 24
3	14, 22, 25, 26, 27, 28, 29, 30
4	17, 23

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 6, 7, 8, 9, 11
2	10, 12, 14, 15, 16, 18, 19, 20, 21
3	13, 22, 24, 25, 26, 27, 28, 29, 30
4	17, 23

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	3	3	1,2 or 3	2 or 3	2 or 3	3	1 or 3	2	3	2	2	2	2	1	3	1 or 3	1
Process Plan (Model 2)	3	2	3	1,2 or 3	3	1,2 or 3	1or 2	3	3	1 or 2	1	1,2 or 3	1 or 3	2	2	3	1	3	1 or 3	2

User Time (Model 1) : 4.33Sec. **User Time** (Model 2) : 4.33Sec.

Objective Function Value (Model 1) : 133892 **Objective Function Value**(Model 2) : 149403

Problem Number 56

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for every part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 9, 10, 13, 15
2	6, 8, 11, 12, 16, 17, 18, 20
3	14, 21, 22, 23
4	19, 24, 25, 26, 27, 28, 29, 30

Model 2

Cell No.	Machines
1	1, 2, 3, 4, 5, 7, 9, 10, 11, 12
2	6, 8, 28, 29,30
3	14, 15, 16, 18, 19, 20
4	13, 17, 21, 22, 23, 24, 25, 26, 27

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2	3	3	3	1,2 or 3	2 or 3	2 or 3	3	1 or 3	2	3	2	1	2	2	2	2 or 3	3	2
Process Plan (Model 2)	2	2	3	1,2 or 3	3	1,2 or 3	1,2 or 3	2 or 3	3	1 or 3	2 or 3	2	1 or 3	2	2	3	2	2 or 3	1 or 3	2

User Time (Model 1) : 3.41Sec. **User Time** (Model 2) : 3.41Sec.

Objective Function Value (Model 1) : 133964 **Objective Function Value**(Model 2) : 149646

Problem Number 57

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for every part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 4, 5, 7, 9, 10, 11, 12, 14
2	3, 6, 8, 24, 25, 27, 29, 30
3	13, 15, 16, 18, 19, 20
4	17, 21, 22, 23, 26, 28

Model 2

Cell No.	Machines
1	1, 2, 4, 5, 7, 9, 10, 11, 12, 14
2	3, 6, 8, 19, 20, 23
3	13, 15, 16, 18, 28
4	17, 21, 22, 23, 24, 25, 26, 27, 29, 30

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2 or 3	3	2 or 3	3	1 or 3	1,2 or3	2 or 3	3	1	2 or 3	2	1,2 or3	2	2	3	2	3	1 or 3	2
Process Plan (Model 2)	3	2 or 3	3	2 or 3	3	1or 3	1,2 or 3	2 or 3	3	1	2 or 3	2	1	2	2	3	2	3	1 or 3	2

User Time (Model 1) : 2.25Sec. **User Time** (Model 2) : 2.25Sec.

Objective Function Value (Model 1) : 133988 **Objective Function Value**(Model 2) : 149819

Problem Number 58

Levels of parameters :

Problem size	High level	(30 machines; 20 parts)
Routing flexibility	High level	(3 process plans for every part)
Range of Processing costs and costs of processing	High level	(2 - 10; 2 - 10)
Range of Production volume of parts	High level	(20 - 48)

Solutions : Model 1

Cell No.	Machines
1	1, 2, 4, 5, 7, 9, 10, 11, 12, 14
2	3, 6, 8, 13, 15, 18, 19, 20
3	16, 17, 21, 22, 23, 27, 29
4	17, 24, 25, 26, 28, 30

Model 2

Cell No.	Machines
1	1, 2, 4, 5, 7, 9, 10, 11, 12, 14
2	3, 6, 8, 13
3	15, 16, 17, 18, 19, 20,
4	21, 22, 23, 24, 25, 26, 27, 28, 29, 30

Parts	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Process Plan (Model 1)	3	2 or 3	3	2 or 3	3	1 or 3	1,2 or 3	2 or 3	3	1	2 or 3	2	1 or 2	2	2	2 or 3	1	3	2	2
Process Plan (Model 2)	3	2 or 3	3	2 or 3	3	1 or 3	1,2 or 3	2 or 3	3	1	2 or 3	2	1	2	2	2 or 3	1	3	1,2 or 3	2

User Time (Model 1) : 1.21Sec. **User Time** (Model 2) : 1.21Sec.

Objective Function Value (Model 1) : 134180 **Objective Function Value**(Model 2) : 152124

Chapter 4

CONCLUSIONS AND SCOPES FOR FURTHER WORK

4.1 Conclusions

There are many techniques available for solving machine grouping problem. These techniques vary on the account of their input, basic approach followed or the objective of grouping. A review of the literature suggests a need of more realistic approach to machine grouping problems. Many of the earlier works have neglected some important considerations. In the present work, an attempt has been made to enlarge the scope of the grouping problem by incorporating more parameters to bring the problem closer to the real life industrial problem. The practical aspects considered are :

- Operation sequence of parts for processing
- Processing time required for a part on machines
- Cost of processing incurred due to processing of parts
- Alternate part routings for every parts
- Material handling costs
- Maximum number of machines can be accommodated in one cell

The resulting grouping problem is formulated as a (0-1) integer programming problem. Subsequently a truncated tree search heuristic, as a solution methodology, is applied to solve this grouping problem. This approach is a greedy approach because it selects only the better nodes at each stage. If the value of truncating parameter is lesser then greediness of this technique is increased and there is more chance to obtain a relatively worse solution.

Several parameters like problem size, routing flexibility, truncating parameters, levels of processing costs and time, and the levels of production volume, have been chosen to investigate the impact on user time, value of objective function and solution.

User time required in solving grouping problem is greatly affected by number of machines, routing flexibility and truncating parameter. Greater number of parts and machines necessitates more computation because it requires sum of the processing costs and the material handling cost for every parts which is incurred due to movement of parts

for processing depending upon the operations and the machines required by those parts. For greater routing flexibility, more computations and comparisons are required which leads to increase the user time. User time also increases with the increase in the truncating parameter because for higher value of truncating parameter, more nodes will be created and explored in getting final solution as compared to that for lesser value of truncating parameter.

Value of the objective function i.e. the sum of processing costs and material handling costs, decreases with increase in the truncating parameter and the routing flexibility. For high value of truncating parameter, more nodes will be explored in getting final solution and there is more chance to obtain better result i.e. lesser sum of processing costs and material handling costs. For higher routing flexibility, there are more alternative paths for processing of parts and hence the chance of obtaining better results increases as compared to that for lesser routing flexibility.

The truncating parameter which determines the quality of solutions and user time should be assigned in such a manner that a balance between these two will be reached at.

Any concrete conclusion cannot be made on the basis of observation regarding impact of the level of production volume range and the level of processing time and cost of processing ranges on the solution because data are not sufficient. For drawing any meaningful conclusion, it is required to have more experimentation for wide range of the parameters value.

The level of routing flexibility affects the solution in every cases of problems. Here process plans of a part are too much differ from each other. Hence, this results in an observation that the level of routing flexibility affects the solution.

For small size problems. it is observed that there is lesser effect of the levels of these attributes (*i.e.* processing time and cost of processing range, routing flexibility, production volume range) as compared to that for large size problems. For small size problems, there is less number of different solutions because lesser number of nodes are formed at the levels of solution procedure and there is more chance of getting the same nodes for different problems as compared to the case of large size problems. Therefore, the number of different solution is less and this results in observation that there is lesser effect of the levels of these attributes on solution for small size problems.

Therefore, the results and their discussions validates the proposed solution

parameter on the solution, the user time and the value of objective function. It is also observed that proposed solution procedure is able to solve the problem in reasonable lesser time.

4.2 Scope for Further Work

Although a lot of research has been done in cellular manufacturing systems, it still remains unexplored. There are so many practical aspects which remain untouched. There are so many economic considerations in practice by considering which we can move more close to real life problem.

For extending this work

- A constraint related with cell machine type that one cell should not consist of more than one machine of same type, can be dropped. For this, one has to consider processing capacity of machines also. Otherwise there is no meaning to solve this problem.
- Actually a cell containing single machine, is not very meaningful. So, by adding an extra constraint that a cell should consist of more than one machine, this problem can be solved. For this case, this problem should be solved as a network flow problem.
- A parametric analysis can be done by varying α and β *i.e.* the material handling cost per unit related with intercell and intracell movements respectively in case of Model 1.

  in case of Model 2, parametric analysis can be done by varying λ and μ *i.e.* ratio of the material handling costs per unit related with intercell and intracell movements and processing costs of succeeding operation respectively.
- Layouts of the cells can be considered. In this case, a simulation study will be easier and more fruitful.
- Stochastic processing time and stochastic demand are also very important in practical environment. For this, again a simulation study will be easier and more fruitful.
- Fixed cost for installation of machines and cells can be considered as additional information for cell formation.

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Appendix

A-1 : Primary Input Data

For the illustration of the formulation and the solution procedure, a problem is taken. In this problem, there are 15 machines and 10 parts. Every part has two separate process plans. There are two copies of machines of type 1 and type 2. Rest of the machines are of different types *i.e.* there is a single copy of machines of each type except for type 1 and type 2. The maximum number of machines that can be accommodated in one cell is 5.. Range for the processing costs and the processing times is 1-5 and 1-5 respectively and the production volume for each part is 10. For singleton type of machines (single copy of own type), the decision variable $X_{p,m}^{r,o}$ is equivalent to the indicator variable $A_{p,m}^{r,o}$.

Material handling costs per unit related with intercell and intracell movement α and β are 4 and 1 respectively as for Model 1.

Concerning Model 2, the ratio of material handling costs and the processing cost of the succeeding operation related with intercell and intracell movement, λ and μ are taken as 0.8 and 0.2 respectively.

Parameters

$M = 15$; $N = 10$; $U^c = 5$; $I = 13$; $R_p = 2$ (for all parts) ;

$B_{1,1}= 1$; $B_{2,2}= 1$; $B_{3,3}= 1$; $B_{4,4}= 1$; $B_{5,5}= 1$; $B_{6,6}= 1$; $B_{7,7}= 1$; $B_{8,8}= 1$;
$B_{9,9}= 1$; $B_{10,10}= 1$; $B_{11,11}= 1$; $B_{12,12}= 1$; $B_{13,13}= 1$; $B_{14,1}= 1$; $B_{15,2}= 1$.

$V_1= 10$; $V_2= 10$; $V_3= 10$; $V_4= 10$; $V_5= 10$; $V_6= 10$; $V_7= 10$; $V_8= 10$;
$V_9= 10$; $V_{10}= 10$; $V_{11}= 10$; $V_{12}= 10$; $V_{13}= 10$; $V_{14}= 10$; $V_{15}= 10$;

Table A1 : Value of $t^{r}_{p,m}$ and $c^{r}_{p,m}$ corresponding to the value of p, r and m

$p \rightarrow$	1		2		3		4		5		6		7		8		9		10	
$r \rightarrow$	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
m $\downarrow$ 1	5			4		5	4	5			4			4	4	3		5	3	
2		5		5	5			4		5			5	3		5				
3	4	4					5		5			4								
4												5	4	5					4	
5											5									3
6																	5	4		4
7																	4			
8			4														3	3		5
9																			5	
10	3		5	3	4					4					5	4				
11						4			4	3			3							
12																				
13		3																		
14	5			4		5	4	5			4			4	4	3		5	3	
15		5		5	5			4		5			5	3		5				

Table A2 : Value of o corresponding to the value of p, r and m.

$p \rightarrow$	1		2		3		4		5		6		7		8		9		10	
$r \rightarrow$	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
m $\downarrow$ 1	1			2		1	2	1			2			2	2	3		1	3	
2		1		1	1			2		1			1	3		1				
3	2	2					1		1			2								
4												1	2	1					2	
5											1									3
6																	1	2		2
7																	2			
8			2														3	3		1
9																			1	
10	3		1	3	2					2					1	2				
11						2			2	3			3							
12																				
13		3																		
14	1			2		1	2	1			2			2	2	3		1	3	
15		1		1	1			2		1			1	3		1				

$A^{r,o}_{p,m} =$ 1 for p, m, r, o when $o \neq 0$ for corresponding to that value of p, r and m.

$O^{1}_{1} = 3$; $O^{2}_{1} = 3$; $O^{1}_{2} = 3$; $O^{2}_{2} = 3$: $O^{1}_{3} = 2$; $O^{2}_{3} = 3$; $O^{1}_{4} = 2$; $O^{2}_{4} = 2$; $O^{1}_{5} = 2$: $O^{2}_{5} = 3$;

$O^{1}_{6} = 2$; $O^{2}_{6} = 2$; $O^{1}_{7} = 3$; $O^{2}_{7} = 3$; $O^{1}_{8} = 2$; $O^{2}_{8} = 3$; $O^{1}_{9} = 3$; $O^{2}_{9} = 3$; $O^{1}_{10} = 3$; $O^{2}_{10} = 3$.

A-2 : Details of Objective Functions and Constraints

A-2.1 *Objective Function :*

A-2.1a *Model 1 :*

To minimize

$$10\left[Z_1^1\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}+Z_1^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_2^1\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}+Z_2^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_3^1\{(5\times5\times1)+(4\times4\times1)\}+Z_3^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_4^1\{(5\times5\times1)+(4\times4\times1)\}+Z_4^2\{(5\times5\times1)+(4\times4\times1)\}\right]+$$

$$10\left[Z_5^1\{(5\times5\times1)+(4\times4\times1)\}+Z_5^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_6^1\{(5\times5\times1)+(4\times4\times1)\}+Z_6^2\{(5\times5\times1)+(4\times4\times1)\}\right]+$$

$$10\left[Z_7^1\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}+Z_7^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_8^1\{(5\times5\times1)+(4\times4\times1)\}+Z_8^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_9^1\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}+Z_9^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_{10}^1\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}+Z_{10}^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\times Z_1^1\times\left[Y_3^1\left(1+3\left(1-X_{1,1}^{1,1}Y_1^1\right)\right)+Y_3^2\left(1+3\left(1-X_{1,1}^{1,1}Y_1^2\right)\right)+Y_3^3\left(1+3\left(1-X_{1,1}^{1,1}Y_1^3\right)\right)+\right.$$
$$Y_3^4\left(1+3\left(1-X_{1,1}^{1,1}Y_1^4\right)\right)+Y_3^5\left(1+3\left(1-X_{1,1}^{1,1}Y_1^5\right)\right)+Y_3^6\left(1+3\left(1-X_{1,1}^{1,1}Y_1^6\right)\right)+Y_3^7\left(1+3\left(1-X_{1,1}^{1,1}Y_1^7\right)\right)+$$
$$Y_3^8\left(1+3\left(1-X_{1,1}^{1,1}Y_1^8\right)\right)+Y_3^9\left(1+3\left(1-X_{1,1}^{1,1}Y_1^9\right)\right)+Y_3^{10}\left(1+3\left(1-X_{1,1}^{1,1}Y_1^{10}\right)\right)+Y_3^{11}\left(1+3\left(1-X_{1,1}^{1,1}Y_1^{11}\right)\right)+$$
$$\left.Y_3^{12}\left(1+3\left(1-X_{1,1}^{1,1}Y_1^{12}\right)\right)+Y_3^{13}\left(1+3\left(1-X_{1,1}^{1,1}Y_1^{13}\right)\right)+Y_3^{14}\left(1+3\left(1-X_{1,1}^{1,1}Y_1^{14}\right)\right)+Y_3^{15}\left(1+3\left(1-X_{1,1}^{1,1}Y_1^{15}\right)\right)\right]$$

$$+10\times Z_1^1\times\left[Y_3^1\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^1\right)\right)+Y_3^2\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^2\right)\right)+Y_3^3\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^3\right)\right)+\right.$$
$$Y_3^4\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^4\right)\right)+Y_3^5\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^5\right)\right)+Y_3^6\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^6\right)\right)+Y_3^7\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^7\right)\right)+$$
$$Y_3^8\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^8\right)\right)+Y_3^9\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^9\right)\right)+Y_3^{10}\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^{10}\right)\right)+Y_3^{11}\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^{11}\right)\right)+$$
$$\left.Y_3^{12}\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^{12}\right)\right)+Y_3^{13}\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^{13}\right)\right)+Y_3^{14}\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^{14}\right)\right)+Y_3^{15}\left(1+3\left(1-X_{1,14}^{1,1}Y_{14}^{15}\right)\right)\right]$$

$$+10\times Z_1^1\times\left[Y_{10}^1\left(1+3\left(1-Y_3^1\right)\right)+Y_{10}^2\left(1+3\left(1-Y_3^2\right)\right)+Y_{10}^3\left(1+3\left(1-Y_3^3\right)\right)+\right.$$

$$Y_{10}^{4}\left(1+3\left(1-Y_{3}^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-Y_{3}^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-Y_{3}^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-Y_{3}^{7}\right)\right)+$$
$$Y_{10}^{8}\left(1+3\left(1-Y_{3}^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-Y_{3}^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-Y_{3}^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-Y_{3}^{11}\right)\right)+$$
$$Y_{10}^{12}\left(1+3\left(1-Y_{3}^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-Y_{3}^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-Y_{3}^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-Y_{3}^{15}\right)\right)\Big]$$

$$+10\times Z_{1}^{2}\times\Big[Y_{3}^{1}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{1}\right)\right)+Y_{3}^{2}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{2}\right)\right)+Y_{3}^{3}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{3}\right)\right)+$$
$$Y_{3}^{4}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{4}\right)\right)+Y_{3}^{5}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{5}\right)\right)+Y_{3}^{6}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{6}\right)\right)+Y_{3}^{7}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{7}\right)\right)+$$
$$Y_{3}^{8}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{8}\right)\right)+Y_{3}^{9}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{9}\right)\right)+Y_{3}^{10}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{10}\right)\right)+Y_{3}^{11}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{11}\right)\right)+$$
$$Y_{3}^{12}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{12}\right)\right)+Y_{3}^{13}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{13}\right)\right)+Y_{3}^{14}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{14}\right)\right)+Y_{3}^{15}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{15}\right)\right)\Big]$$

$$+10\times Z_{1}^{2}\times\Big[Y_{3}^{1}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{1}\right)\right)+Y_{3}^{2}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{2}\right)\right)+Y_{3}^{3}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{3}\right)\right)+$$
$$Y_{3}^{4}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{4}\right)\right)+Y_{3}^{5}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{5}\right)\right)+Y_{3}^{6}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{6}\right)\right)+Y_{3}^{7}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{7}\right)\right)+$$
$$Y_{3}^{8}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{8}\right)\right)+Y_{3}^{9}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{9}\right)\right)+Y_{3}^{10}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{10}\right)\right)+Y_{3}^{11}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{11}\right)\right)+$$
$$Y_{3}^{12}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{12}\right)\right)+Y_{3}^{13}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{13}\right)\right)+Y_{3}^{14}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{14}\right)\right)+Y_{3}^{15}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_{1}^{2}\times\Big[Y_{13}^{1}\left(1+3\left(1-Y_{3}^{1}\right)\right)+Y_{13}^{2}\left(1+3\left(1-Y_{3}^{2}\right)\right)+Y_{13}^{3}\left(1+3\left(1-Y_{3}^{3}\right)\right)+$$
$$Y_{13}^{4}\left(1+3\left(1-Y_{3}^{4}\right)\right)+Y_{13}^{5}\left(1+3\left(1-Y_{3}^{5}\right)\right)+Y_{13}^{6}\left(1+3\left(1-Y_{3}^{6}\right)\right)+Y_{13}^{7}\left(1+3\left(1-Y_{3}^{7}\right)\right)+$$
$$Y_{13}^{8}\left(1+3\left(1-Y_{3}^{8}\right)\right)+Y_{13}^{9}\left(1+3\left(1-Y_{3}^{9}\right)\right)+Y_{13}^{10}\left(1+3\left(1-Y_{3}^{10}\right)\right)+Y_{13}^{11}\left(1+3\left(1-Y_{3}^{11}\right)\right)+$$
$$Y_{13}^{12}\left(1+3\left(1-Y_{3}^{12}\right)\right)+Y_{13}^{13}\left(1+03\left(1-Y_{3}^{13}\right)\right)+Y_{13}^{14}\left(1+3\left(1-Y_{3}^{14}\right)\right)+Y_{13}^{15}\left(1+3\left(1-Y_{3}^{15}\right)\right)\Big]$$

$$+10\times Z_{2}^{1}\times\Big[Y_{8}^{1}\left(1+3\left(1-Y_{10}^{1}\right)\right)+Y_{8}^{2}\left(1+3\left(1-Y_{10}^{2}\right)\right)+Y_{8}^{3}\left(1+3\left(1-Y_{10}^{3}\right)\right)+$$
$$Y_{8}^{4}\left(1+3\left(1-Y_{10}^{4}\right)\right)+Y_{8}^{5}\left(1+3\left(1-Y_{10}^{5}\right)\right)+Y_{8}^{6}\left(1+3\left(1-Y_{10}^{6}\right)\right)+Y_{8}^{7}\left(1+3\left(1-Y_{10}^{7}\right)\right)+$$
$$Y_{8}^{8}\left(1+3\left(1-Y_{10}^{8}\right)\right)+Y_{8}^{9}\left(1+3\left(1-Y_{10}^{9}\right)\right)+Y_{8}^{10}\left(1+3\left(1-Y_{10}^{10}\right)\right)+Y_{8}^{11}\left(1+3\left(1-Y_{10}^{11}\right)\right)+$$
$$Y_{8}^{12}\left(1+3\left(1-Y_{10}^{12}\right)\right)+Y_{8}^{13}\left(1+3\left(1-Y_{10}^{13}\right)\right)+Y_{8}^{14}\left(1+3\left(1-Y_{10}^{14}\right)\right)+Y_{8}^{15}\left(1+3\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_{2}^{2}\times X_{2,1}^{2,2}\Big[Y_{1}^{1}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{1}\right)\right)+Y_{1}^{2}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{2}\right)\right)+Y_{1}^{3}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{3}\right)\right)+$$
$$Y_{1}^{4}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{4}\right)\right)+Y_{1}^{5}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{5}\right)\right)+Y_{1}^{6}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{6}\right)\right)+Y_{1}^{7}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{7}\right)\right)+$$
$$Y_{1}^{8}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{8}\right)\right)+Y_{1}^{9}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{9}\right)\right)+Y_{1}^{10}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{10}\right)\right)+Y_{1}^{11}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{11}\right)\right)+$$
$$Y_{1}^{12}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{12}\right)\right)+Y_{1}^{13}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{13}\right)\right)+Y_{1}^{14}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{14}\right)\right)+Y_{1}^{15}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{15}\right)\right)\Big]$$

$$+10\times Z_{2}^{2}\times X_{2,14}^{2,2}\Big[Y_{14}^{1}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{1}\right)\right)+Y_{14}^{2}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{2}\right)\right)+Y_{14}^{3}\left(1+3\left(1-X_{1,2}^{2,1}Y_{2}^{3}\right)\right)+$$

$$Y_{14}^{4}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{4}\right)\right)+Y_{14}^{5}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{5}\right)\right)+Y_{14}^{6}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{6}\right)\right)+Y_{14}^{7}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{7}\right)\right)+$$

$$Y_{14}^{8}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{8}\right)\right)+Y_{14}^{9}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{9}\right)\right)+Y_{14}^{10}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{10}\right)\right)+Y_{14}^{11}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{11}\right)\right)+$$

$$Y_{14}^{12}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{12}\right)\right)+Y_{14}^{13}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{13}\right)\right)+Y_{14}^{14}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{14}\right)\right)+Y_{14}^{15}\left(1+3\left(1-X_{1,2}^{2,1}Y_2^{15}\right)\right)\Big]$$

$$+10\times Z_2^2\times X_{2,1}^{2,2}\Big[Y_1^{1}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{1}\right)\right)+Y_1^{2}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{2}\right)\right)+Y_1^{3}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{3}\right)\right)+$$

$$Y_1^{4}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{4}\right)\right)+Y_1^{5}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{5}\right)\right)+Y_1^{6}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{6}\right)\right)+Y_1^{7}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{7}\right)\right)+$$

$$Y_1^{8}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{8}\right)\right)+Y_1^{9}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{9}\right)\right)+Y_1^{10}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{10}\right)\right)+Y_1^{11}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{11}\right)\right)+$$

$$Y_1^{12}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{12}\right)\right)+Y_1^{13}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{13}\right)\right)+Y_1^{14}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{14}\right)\right)+Y_1^{15}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_2^2\times X_{2,14}^{2,2}\Big[Y_{14}^{1}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{1}\right)\right)+Y_{14}^{2}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{2}\right)\right)+Y_{14}^{3}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{3}\right)\right)+$$

$$Y_{14}^{4}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{4}\right)\right)+Y_{14}^{5}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{5}\right)\right)+Y_{14}^{6}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{6}\right)\right)+Y_{14}^{7}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{7}\right)\right)+$$

$$Y_{14}^{8}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{8}\right)\right)+Y_{14}^{9}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{9}\right)\right)+Y_{14}^{10}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{10}\right)\right)+Y_{14}^{11}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{11}\right)\right)+$$

$$Y_{14}^{12}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{12}\right)\right)+Y_{14}^{13}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{13}\right)\right)+Y_{14}^{14}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{14}\right)\right)+Y_{14}^{15}\left(1+3\left(1-X_{1,15}^{2,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_2^2\times\Big[Y_{10}^{1}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{1}\right)\right)+Y_{10}^{2}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{2}\right)\right)+Y_{10}^{3}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{3}\right)\right)+$$

$$Y_{10}^{4}\left(1+3\left(1-X_{2,1}^{2,2}X_{1,1}^{1,1}Y_1^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{7}\right)\right)+$$

$$Y_{10}^{8}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{11}\right)\right)+$$

$$Y_{10}^{12}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-X_{2,1}^{2,2}Y_1^{15}\right)\right)\Big]$$

$$+10\times Z_2^2\times\Big[Y_{10}^{1}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{1}\right)\right)+Y_{10}^{2}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{2}\right)\right)+Y_{10}^{3}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{3}\right)\right)+$$

$$Y_{10}^{4}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{7}\right)\right)+$$

$$Y_{10}^{8}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{11}\right)\right)+$$

$$Y_{10}^{12}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-X_{2,14}^{2,2}Y_{14}^{15}\right)\right)\Big]$$

$$+10\times Z_3^1\times\Big[Y_{10}^{1}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{1}\right)\right)+Y_{10}^{2}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{2}\right)\right)+Y_{10}^{3}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{3}\right)\right)+$$

$$Y_{10}^{4}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{7}\right)\right)+$$

$$Y_{10}^{8}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{11}\right)\right)+$$

$$Y_{10}^{12}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-X_{3,2}^{1,1}Y_2^{15}\right)\right)\Big]$$

$$+10\times Z_3^1\times\Big[Y_{10}^{1}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{1}\right)\right)+Y_{10}^{2}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{2}\right)\right)+Y_{10}^{3}\left(1+1\left(1-X_{3,15}^{1,1}Y_{15}^{3}\right)\right)+$$

$$Y_{10}^{4}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{7}\right)\right)+$$
$$Y_{10}^{8}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{11}\right)\right)+$$
$$Y_{10}^{12}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-X_{3,15}^{1,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_3^2\times\Big[Y_{11}^{1}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{1}\right)\right)+Y_{11}^{2}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{2}\right)\right)+Y_{11}^{3}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{3}\right)\right)+$$

$$Y_{11}^{4}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{4}\right)\right)+Y_{11}^{5}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{5}\right)\right)+Y_{11}^{6}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{6}\right)\right)+Y_{11}^{7}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{7}\right)\right)+$$
$$Y_{11}^{8}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{8}\right)\right)+Y_{11}^{9}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{9}\right)\right)+Y_{11}^{10}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{10}\right)\right)+Y_{11}^{11}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{11}\right)\right)+$$
$$Y_{11}^{12}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{12}\right)\right)+Y_{11}^{13}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{13}\right)\right)+Y_{11}^{14}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{14}\right)\right)+Y_{11}^{15}\left(1+3\left(1-X_{3,1}^{2,1}Y_1^{15}\right)\right)\Big]$$

$$+10\times Z_3^2\times\Big[Y_{11}^{1}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{1}\right)\right)+Y_{11}^{2}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{2}\right)\right)+Y_{11}^{3}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{3}\right)\right)+$$

$$Y_{11}^{4}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{4}\right)\right)+Y_{11}^{5}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{5}\right)\right)+Y_{11}^{6}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{6}\right)\right)+Y_{11}^{7}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{7}\right)\right)+$$
$$Y_{11}^{8}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{8}\right)\right)+Y_{11}^{9}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{9}\right)\right)+Y_{11}^{10}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{10}\right)\right)+Y_{11}^{11}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{11}\right)\right)+$$
$$Y_{11}^{12}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{12}\right)\right)+Y_{11}^{13}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{13}\right)\right)+Y_{11}^{14}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{14}\right)\right)+Y_{11}^{15}\left(1+3\left(1-X_{3,14}^{2,1}Y_{14}^{15}\right)\right)\Big]$$

$$+10\times Z_4^1\times X_{4,1}^{1,2}\Big[Y_1^{1}\left(1+3\left(1-Y_3^{1}\right)\right)+Y_1^{2}\left(1+3\left(1-Y_3^{2}\right)\right)+Y_1^{3}\left(1+3\left(1-Y_3^{3}\right)\right)+$$

$$Y_1^{4}\left(1+3\left(1-Y_3^{4}\right)\right)+Y_1^{5}\left(1+3\left(1-Y_3^{5}\right)\right)+Y_1^{6}\left(1+3\left(1-Y_3^{6}\right)\right)+Y_1^{7}\left(1+3\left(1-Y_3^{7}\right)\right)+$$
$$Y_1^{8}\left(1+3\left(1-Y_3^{8}\right)\right)+Y_1^{9}\left(1+3\left(1-Y_3^{9}\right)\right)+Y_1^{10}\left(1+3\left(1-Y_3^{10}\right)\right)+Y_1^{11}\left(1+3\left(1-Y_3^{11}\right)\right)+$$
$$Y_1^{12}\left(1+3\left(1-Y_3^{12}\right)\right)+Y_1^{13}\left(1+3\left(1-Y_3^{13}\right)\right)+Y_1^{14}\left(1+3\left(1-Y_3^{14}\right)\right)+Y_1^{15}\left(1+3\left(1-Y_3^{15}\right)\right)\Big]$$

$$+10\times Z_4^1\times X_{4,14}^{1,2}\Big[Y_{14}^{1}\left(1+3\left(1-Y_3^{1}\right)\right)+Y_{14}^{2}\left(1+3\left(1-Y_3^{2}\right)\right)+Y_{14}^{3}\left(1+3\left(1-Y_3^{3}\right)\right)+$$

$$Y_{14}^{4}\left(1+3\left(1-Y_3^{4}\right)\right)+Y_{14}^{5}\left(1+3\left(1-Y_3^{5}\right)\right)+Y_{14}^{6}\left(1+3\left(1-Y_3^{6}\right)\right)+Y_{14}^{7}\left(1+3\left(1-Y_3^{7}\right)\right)+$$
$$Y_{14}^{8}\left(1+3\left(1-Y_3^{8}\right)\right)+Y_{14}^{9}\left(1+3\left(1-Y_3^{9}\right)\right)+Y_{14}^{10}\left(1+3\left(1-Y_3^{10}\right)\right)+Y_{14}^{11}\left(1+3\left(1-Y_3^{11}\right)\right)+$$
$$Y_{14}^{12}\left(1+3\left(1-Y_3^{12}\right)\right)+Y_{14}^{13}\left(1+3\left(1-Y_3^{13}\right)\right)+Y_{14}^{14}\left(1+3\left(1-Y_3^{14}\right)\right)+Y_{14}^{15}\left(1+3\left(1-Y_3^{15}\right)\right)\Big]$$

$$+10\times Z_4^2\times X_{4,2}^{2,2}\Big[Y_2^{1}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{1}\right)\right)+Y_2^{2}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{2}\right)\right)+Y_2^{3}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{3}\right)\right)+$$

$$Y_2^{4}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{4}\right)\right)+Y_2^{5}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{5}\right)\right)+Y_2^{6}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{6}\right)\right)+Y_2^{7}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{7}\right)\right)+$$
$$Y_2^{8}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{8}\right)\right)+Y_2^{9}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{9}\right)\right)+Y_2^{10}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{10}\right)\right)+Y_2^{11}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{11}\right)\right)+$$
$$Y_2^{12}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{12}\right)\right)+Y_2^{13}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{13}\right)\right)+Y_2^{14}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{14}\right)\right)+Y_2^{15}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{15}\right)\right)\Big]$$

$$+10\times Z_4^2\times X_{4,15}^{2,2}\Big[Y_{15}^{1}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{1}\right)\right)+Y_{15}^{2}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{2}\right)\right)+Y_{15}^{3}\left(1+3\left(1-X_{4,1}^{2,1}Y_1^{3}\right)\right)+$$

$$Y_{15}^{4}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{4}\right)\right)+Y_{15}^{5}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{5}\right)\right)+Y_{15}^{6}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{6}\right)\right)+Y_{15}^{7}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{7}\right)\right)+$$
$$Y_{15}^{8}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{8}\right)\right)+Y_{15}^{9}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{9}\right)\right)+Y_{15}^{10}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{10}\right)\right)+Y_{15}^{11}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{11}\right)\right)+$$
$$Y_{15}^{12}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{12}\right)\right)+Y_{15}^{13}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{13}\right)\right)+Y_{15}^{14}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{14}\right)\right)+Y_{15}^{15}\left(1+3\left(1-X_{4,1}^{2,1}Y_{1}^{15}\right)\right)\Big]$$

$$+10\times Z_{4}^{2}\times X_{4,2}^{2,2}\Big[Y_{2}^{1}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{1}\right)\right)+Y_{2}^{2}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{2}\right)\right)+Y_{2}^{3}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{3}\right)\right)+$$

$$Y_{2}^{4}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{4}\right)\right)+Y_{2}^{5}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{5}\right)\right)+Y_{2}^{6}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{6}\right)\right)+Y_{2}^{7}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{7}\right)\right)+$$
$$Y_{2}^{8}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{8}\right)\right)+Y_{2}^{9}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{9}\right)\right)+Y_{2}^{10}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{10}\right)\right)+Y_{2}^{11}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{11}\right)\right)+$$
$$Y_{2}^{12}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{12}\right)\right)+Y_{2}^{13}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{13}\right)\right)+Y_{2}^{14}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{14}\right)\right)+Y_{2}^{15}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{15}\right)\right)\Big]$$

$$+10\times Z_{4}^{2}\times X_{4,15}^{2,2}\Big[Y_{15}^{1}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{1}\right)\right)+Y_{15}^{2}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{2}\right)\right)+Y_{15}^{3}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{3}\right)\right)+$$

$$Y_{15}^{4}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{4}\right)\right)+Y_{15}^{5}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{5}\right)\right)+Y_{15}^{6}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{6}\right)\right)+Y_{15}^{7}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{7}\right)\right)+$$
$$Y_{15}^{8}\left(1+2\left(1-X_{4,14}^{2,1}Y_{14}^{8}\right)\right)+Y_{15}^{9}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{9}\right)\right)+Y_{15}^{10}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{10}\right)\right)+Y_{15}^{11}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{11}\right)\right)+$$
$$Y_{15}^{12}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{12}\right)\right)+Y_{15}^{13}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{13}\right)\right)+Y_{15}^{14}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{14}\right)\right)+Y_{15}^{15}\left(1+3\left(1-X_{4,14}^{2,1}Y_{14}^{15}\right)\right)\Big]$$

$$+10\times Z_{5}^{1}\times$$

$$\Big[Y_{11}^{1}\left(1+3\left(1-Y_{3}^{1}\right)\right)+Y_{11}^{2}\left(1+3\left(1-Y_{3}^{2}\right)\right)+Y_{11}^{3}\left(1+3\left(1-Y_{3}^{3}\right)\right)+$$
$$Y_{11}^{4}\left(1+3\left(1-Y_{3}^{4}\right)\right)+Y_{11}^{5}\left(1+3\left(1-Y_{3}^{5}\right)\right)+Y_{11}^{6}\left(1+3\left(1-Y_{3}^{6}\right)\right)+Y_{11}^{7}\left(1+3\left(1-Y_{3}^{7}\right)\right)+$$
$$Y_{11}^{8}\left(1+3\left(1-Y_{3}^{8}\right)\right)+Y_{11}^{9}\left(1+3\left(1-Y_{3}^{9}\right)\right)+Y_{11}^{10}\left(1+3\left(1-Y_{3}^{10}\right)\right)+Y_{11}^{11}\left(1+3\left(1-Y_{3}^{11}\right)\right)+$$
$$Y_{11}^{12}\left(1+3\left(1-Y_{3}^{12}\right)\right)+Y_{11}^{13}\left(1+3\left(1-Y_{3}^{13}\right)\right)+Y_{11}^{14}\left(1+3\left(1-Y_{3}^{14}\right)\right)+Y_{11}^{15}\left(1+3\left(1-Y_{3}^{15}\right)\right)\Big]$$

$$+10\times Z_{5}^{2}\times\Big[Y_{10}^{1}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{1}\right)\right)+Y_{10}^{2}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{2}\right)\right)+Y_{10}^{3}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{3}\right)\right)+$$

$$Y_{10}^{4}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{7}\right)\right)+$$
$$Y_{10}^{8}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{11}\right)\right)+$$
$$Y_{10}^{12}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-X_{5,2}^{2,1}Y_{2}^{15}\right)\right)\Big]$$

$$+10\times Z_{5}^{2}\times\Big[Y_{10}^{1}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{1}\right)\right)+Y_{10}^{2}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{2}\right)\right)+Y_{10}^{3}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{3}\right)\right)+$$

$$Y_{10}^{4}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{7}\right)\right)+$$
$$Y_{10}^{8}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{11}\right)\right)+$$
$$Y_{10}^{12}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-X_{5,15}^{2,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_{5}^{2}\times\Big[Y_{11}^{1}\left(1+3\left(1-Y_{10}^{1}\right)\right)+Y_{11}^{2}\left(1+3\left(1-Y_{10}^{2}\right)\right)+Y_{11}^{3}\left(1+3\left(1-Y_{10}^{3}\right)\right)+$$

$$Y_{11}^{4}\left(1+3\left(1-Y_{10}^{4}\right)\right)+Y_{11}^{5}\left(1+3\left(1-Y_{10}^{5}\right)\right)+Y_{11}^{6}\left(1+3\left(1-Y_{10}^{6}\right)\right)+Y_{11}^{7}\left(1+3\left(1-Y_{10}^{7}\right)\right)+$$
$$Y_{11}^{8}\left(1+3\left(1-Y_{10}^{8}\right)\right)+Y_{11}^{9}\left(1+3\left(1-Y_{10}^{9}\right)\right)+Y_{11}^{10}\left(1+3\left(1-Y_{10}^{10}\right)\right)+Y_{11}^{11}\left(1+3\left(1-Y_{10}^{11}\right)\right)+$$
$$Y_{11}^{12}\left(1+3\left(1-Y_{10}^{12}\right)\right)+Y_{11}^{13}\left(1+3\left(1-Y_{10}^{13}\right)\right)+Y_{11}^{14}\left(1+3\left(1-Y_{10}^{14}\right)\right)+Y_{11}^{15}\left(1+3\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_{6}^{1}\times X_{6,1}^{1,2}\Big[Y_{1}^{1}\left(1+3\left(1-Y_{5}^{1}\right)\right)+Y_{1}^{2}\left(1+3\left(1-Y_{5}^{2}\right)\right)+Y_{1}^{3}\left(1+3\left(1-Y_{5}^{3}\right)\right)+$$

$$Y_{1}^{4}\left(1+3\left(1-Y_{5}^{4}\right)\right)+Y_{1}^{5}\left(1+3\left(1-Y_{5}^{5}\right)\right)+Y_{1}^{6}\left(1+3\left(1-Y_{5}^{6}\right)\right)+Y_{1}^{7}\left(1+3\left(1-Y_{5}^{7}\right)\right)+$$
$$Y_{1}^{8}\left(1+3\left(1-Y_{5}^{8}\right)\right)+Y_{1}^{9}\left(1+3\left(1-Y_{5}^{9}\right)\right)+Y_{1}^{10}\left(1+3\left(1-Y_{5}^{10}\right)\right)+Y_{1}^{11}\left(1+3\left(1-Y_{5}^{11}\right)\right)+$$
$$Y_{1}^{12}\left(1+3\left(1-Y_{5}^{12}\right)\right)+Y_{1}^{13}\left(1+3\left(1-Y_{5}^{13}\right)\right)+Y_{1}^{14}\left(1+3\left(1-Y_{5}^{14}\right)\right)+Y_{1}^{15}\left(1+3\left(1-Y_{5}^{15}\right)\right)\Big]$$

$$+10\times Z_{6}^{1}\times X_{6,14}^{1,2}\Big[Y_{14}^{1}\left(1+3\left(1-Y_{5}^{1}\right)\right)+Y_{14}^{2}\left(1+3\left(1-Y_{5}^{2}\right)\right)+Y_{14}^{3}\left(1+3\left(1-Y_{5}^{3}\right)\right)+$$

$$Y_{14}^{4}\left(1+3\left(1-Y_{5}^{4}\right)\right)+Y_{14}^{5}\left(1+3\left(1-Y_{5}^{5}\right)\right)+Y_{14}^{6}\left(1+3\left(1-Y_{5}^{6}\right)\right)+Y_{14}^{7}\left(1+3\left(1-Y_{5}^{7}\right)\right)+$$
$$Y_{14}^{8}\left(1+3\left(1-Y_{5}^{8}\right)\right)+Y_{14}^{9}\left(1+3\left(1-Y_{5}^{9}\right)\right)+Y_{14}^{10}\left(1+3\left(1-Y_{5}^{10}\right)\right)+Y_{14}^{11}\left(1+3\left(1-Y_{5}^{11}\right)\right)+$$
$$Y_{14}^{12}\left(1+3\left(1-Y_{5}^{12}\right)\right)+Y_{14}^{13}\left(1+3\left(1-Y_{5}^{13}\right)\right)+Y_{14}^{14}\left(1+3\left(1-Y_{5}^{14}\right)\right)+Y_{14}^{15}\left(1+3\left(1-Y_{5}^{15}\right)\right)\Big]$$

$$+10\times Z_{6}^{2}\times$$

$$\Big[Y_{3}^{1}\left(1+3\left(1-Y_{4}^{1}\right)\right)+Y_{3}^{2}\left(1+3\left(1-Y_{4}^{2}\right)\right)+Y_{3}^{3}\left(1+3\left(1-Y_{4}^{3}\right)\right)+$$
$$Y_{3}^{4}\left(1+3\left(1-Y_{4}^{4}\right)\right)+Y_{3}^{5}\left(1+3\left(1-Y_{4}^{5}\right)\right)+Y_{3}^{6}\left(1+3\left(1-Y_{4}^{6}\right)\right)+Y_{3}^{7}\left(1+3\left(1-Y_{4}^{7}\right)\right)+$$
$$Y_{3}^{8}\left(1+3\left(1-Y_{4}^{8}\right)\right)+Y_{3}^{9}\left(1+3\left(1-Y_{4}^{9}\right)\right)+Y_{3}^{10}\left(1+3\left(1-Y_{4}^{10}\right)\right)+Y_{3}^{11}\left(1+3\left(1-Y_{4}^{11}\right)\right)+$$
$$Y_{3}^{12}\left(1+3\left(1-Y_{4}^{12}\right)\right)+Y_{3}^{13}\left(1+3\left(1-Y_{4}^{13}\right)\right)+Y_{3}^{14}\left(1+3\left(1-Y_{4}^{14}\right)\right)+Y_{3}^{15}\left(1+3\left(1-Y_{4}^{15}\right)\right)\Big]$$

$$+10\times Z_{7}^{1}\times\Big[Y_{4}^{1}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{1}\right)\right)+Y_{4}^{2}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{2}\right)\right)+Y_{4}^{3}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{3}\right)\right)+$$

$$Y_{4}^{4}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{4}\right)\right)+Y_{4}^{5}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{5}\right)\right)+Y_{4}^{6}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{6}\right)\right)+Y_{4}^{7}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{7}\right)\right)+$$
$$Y_{4}^{8}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{8}\right)\right)+Y_{4}^{9}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{9}\right)\right)+Y_{4}^{10}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{10}\right)\right)+Y_{4}^{11}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{11}\right)\right)+$$
$$Y_{4}^{12}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{12}\right)\right)+Y_{4}^{13}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{13}\right)\right)+Y_{4}^{14}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{14}\right)\right)+Y_{4}^{15}\left(1+3\left(1-X_{7,2}^{1,1}Y_{2}^{15}\right)\right)\Big]$$

$$+10\times Z_{7}^{1}\times\Big[Y_{4}^{1}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{1}\right)\right)+Y_{4}^{2}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{2}\right)\right)+Y_{4}^{3}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{3}\right)\right)+$$

$$Y_{4}^{4}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{4}\right)\right)+Y_{4}^{5}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{5}\right)\right)+Y_{4}^{6}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{6}\right)\right)+Y_{4}^{7}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{7}\right)\right)+$$
$$Y_{4}^{8}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{8}\right)\right)+Y_{4}^{9}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{9}\right)\right)+Y_{4}^{10}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{10}\right)\right)+Y_{4}^{11}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{11}\right)\right)+$$
$$Y_{4}^{12}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{12}\right)\right)+Y_{4}^{13}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{13}\right)\right)+Y_{4}^{14}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{14}\right)\right)+Y_{4}^{15}\left(1+3\left(1-X_{7,15}^{1,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_{7}^{1}\times\Big[Y_{11}^{1}\left(1+3\left(1-Y_{4}^{1}\right)\right)+Y_{11}^{2}\left(1+3\left(1-Y_{4}^{2}\right)\right)+Y_{11}^{3}\left(1+3\left(1-Y_{4}^{3}\right)\right)+$$

$$Y_{11}^{4}\left(1+3\left(1-Y_4^{4}\right)\right)+Y_{11}^{5}\left(1+3\left(1-Y_4^{5}\right)\right)+Y_{11}^{6}\left(1+3\left(1-Y_4^{6}\right)\right)+Y_{11}^{7}\left(1+3\left(1-Y_4^{7}\right)\right)+$$

$$Y_{11}^{8}\left(1+3\left(1-Y_4^{8}\right)\right)+Y_{11}^{9}\left(1+3\left(1-Y_4^{9}\right)\right)+Y_{11}^{10}\left(1+3\left(1-Y_4^{10}\right)\right)+Y_{11}^{11}\left(1+3\left(1-Y_4^{11}\right)\right)+$$

$$Y_{11}^{12}\left(1+3\left(1-Y_4^{12}\right)\right)+Y_{11}^{13}\left(1+3\left(1-Y_4^{13}\right)\right)+Y_{11}^{14}\left(1+3\left(1-Y_4^{14}\right)\right)+Y_{11}^{15}\left(1+3\left(1-Y_4^{15}\right)\right)\Big]$$

$$+10\times Z_7^2\times X_{7,1}^{2,2}\Big[Y_1^{1}\left(1+3\left(1-Y_4^{1}\right)\right)+Y_1^{2}\left(1+3\left(1-Y_4^{2}\right)\right)+Y_1^{3}\left(1+3\left(1-Y_4^{3}\right)\right)+$$

$$Y_1^{4}\left(1+3\left(1-Y_4^{4}\right)\right)+Y_1^{5}\left(1+3\left(1-Y_4^{5}\right)\right)+Y_1^{6}\left(1+3\left(1-Y_4^{6}\right)\right)+Y_1^{7}\left(1+3\left(1-Y_4^{7}\right)\right)+$$

$$Y_1^{8}\left(1+3\left(1-Y_4^{8}\right)\right)+Y_1^{9}\left(1+3\left(1-Y_4^{9}\right)\right)+Y_1^{10}\left(1+3\left(1-Y_4^{10}\right)\right)+Y_1^{11}\left(1+3\left(1-Y_4^{11}\right)\right)+$$

$$Y_1^{12}\left(1+3\left(1-Y_4^{12}\right)\right)+Y_1^{13}\left(1+3\left(1-Y_4^{13}\right)\right)+Y_1^{14}\left(1+3\left(1-Y_4^{14}\right)\right)+Y_1^{15}\left(1+3\left(1-Y_4^{15}\right)\right)\Big]$$

$$+10\times Z_7^2\times X_{7,14}^{2,2}\Big[Y_{14}^{1}\left(1+3\left(1-Y_4^{1}\right)\right)+Y_{14}^{2}\left(1+3\left(1-Y_4^{2}\right)\right)+Y_{14}^{3}\left(1+3\left(1-Y_4^{3}\right)\right)+$$

$$Y_{14}^{4}\left(1+3\left(1-Y_4^{4}\right)\right)+Y_{14}^{5}\left(1+3\left(1-Y_4^{5}\right)\right)+Y_{14}^{6}\left(1+3\left(1-Y_4^{6}\right)\right)+Y_{14}^{7}\left(1+3\left(1-Y_4^{7}\right)\right)+$$

$$Y_{14}^{8}\left(1+3\left(1-Y_4^{8}\right)\right)+Y_{14}^{9}\left(1+3\left(1-Y_4^{9}\right)\right)+Y_{14}^{10}\left(1+3\left(1-Y_4^{10}\right)\right)+Y_{14}^{11}\left(1+3\left(1-Y_4^{11}\right)\right)+$$

$$Y_{14}^{12}\left(1+3\left(1-Y_4^{12}\right)\right)+Y_{14}^{13}\left(1+3\left(1-Y_4^{13}\right)\right)+Y_{14}^{14}\left(1+3\left(1-Y_4^{14}\right)\right)+Y_{14}^{15}\left(1+3\left(1-Y_4^{15}\right)\right)\Big]$$

$$+10\times Z_7^2\times X_{7,2}^{2,3}\Big[Y_2^{1}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{1}\right)\right)+Y_2^{2}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{2}\right)\right)+Y_2^{3}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{3}\right)\right)+$$

$$Y_2^{4}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{4}\right)\right)+Y_2^{5}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{5}\right)\right)+Y_2^{6}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{6}\right)\right)+Y_2^{7}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{7}\right)\right)+$$

$$Y_2^{8}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{8}\right)\right)+Y_2^{9}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{9}\right)\right)+Y_2^{10}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{10}\right)\right)+Y_2^{11}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{11}\right)\right)+$$

$$Y_2^{12}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{12}\right)\right)+Y_2^{13}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{13}\right)\right)+Y_2^{14}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{14}\right)\right)+Y_2^{15}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{15}\right)\right)\Big]+10$$

$$\times Z_7^2\times X_{7,15}^{2,3}\Big[Y_{15}^{1}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{1}\right)\right)+Y_{15}^{2}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{2}\right)\right)+Y_{15}^{3}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{3}\right)\right)+$$

$$Y_{15}^{4}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{4}\right)\right)+Y_{15}^{5}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{5}\right)\right)+Y_{15}^{6}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{6}\right)\right)+Y_{15}^{7}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{7}\right)\right)+$$

$$Y_{15}^{8}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{8}\right)\right)+Y_{15}^{9}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{9}\right)\right)+Y_{15}^{10}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{10}\right)\right)+Y_{15}^{11}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{11}\right)\right)+$$

$$Y_{15}^{12}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{12}\right)\right)+Y_{15}^{13}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{13}\right)\right)+Y_{15}^{14}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{14}\right)\right)+Y_{15}^{15}\left(1+3\left(1-X_{7,1}^{2,2}Y_1^{15}\right)\right)\Big]$$

$$+10\times Z_7^2\times X_{7,2}^{2,3}\Big[Y_2^{1}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{1}\right)\right)+Y_2^{2}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{2}\right)\right)+Y_2^{3}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{3}\right)\right)+$$

$$Y_2^{4}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{4}\right)\right)+Y_2^{5}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{5}\right)\right)+Y_2^{6}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{6}\right)\right)+Y_2^{7}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{7}\right)\right)+$$

$$Y_2^{8}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{8}\right)\right)+Y_2^{9}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{9}\right)\right)+Y_2^{10}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{10}\right)\right)+Y_2^{11}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{11}\right)\right)+$$

$$Y_2^{12}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{12}\right)\right)+Y_2^{13}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{13}\right)\right)+Y_2^{14}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{14}\right)\right)+Y_2^{15}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{15}\right)\right)\Big]+$$

$$10\times Z_7^2\times X_{7,15}^{2,3}\Big[Y_{15}^{1}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{1}\right)\right)+Y_{15}^{2}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{2}\right)\right)+Y_{15}^{3}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{3}\right)\right)+$$

$$Y_{15}^{4}\left(1+3\left(1-Y_{14}^{4}\right)\right)+Y_{15}^{5}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{5}\right)\right)+Y_{15}^{6}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{6}\right)\right)+Y_{15}^{7}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{7}\right)\right)+$$

$$Y_{15}^{8}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{8}\right)\right)+Y_{15}^{9}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{9}\right)\right)+Y_{15}^{10}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{10}\right)\right)+Y_{15}^{11}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{11}\right)\right)+$$

$$Y_{15}^{12}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{12}\right)\right)+Y_{15}^{13}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{13}\right)\right)+Y_{15}^{14}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{14}\right)\right)+Y_{15}^{15}\left(1+3\left(1-X_{7,14}^{2,2}Y_{14}^{15}\right)\right)\Big]$$

$$+10\times Z_{8}^{1}\times X_{8,1}^{1,2}\Big[Y_{1}^{1}\left(1+3\left(1-Y_{10}^{1}\right)\right)+Y_{1}^{2}\left(1+3\left(1-Y_{10}^{2}\right)\right)+Y_{1}^{3}\left(1+3\left(1-Y_{10}^{3}\right)\right)+$$

$$Y_{1}^{4}\left(1+3\left(1-Y_{10}^{4}\right)\right)+Y_{1}^{5}\left(1+3\left(1-Y_{10}^{5}\right)\right)+Y_{1}^{6}\left(1+3\left(1-Y_{10}^{6}\right)\right)+Y_{1}^{7}\left(1+3\left(1-Y_{10}^{7}\right)\right)+$$

$$Y_{1}^{8}\left(1+3\left(1-Y_{10}^{8}\right)\right)+Y_{1}^{9}\left(1+3\left(1-Y_{10}^{9}\right)\right)+Y_{1}^{10}\left(1+3\left(1-Y_{10}^{10}\right)\right)+Y_{1}^{11}\left(1+3\left(1-Y_{10}^{11}\right)\right)+$$

$$Y_{1}^{12}\left(1+3\left(1-Y_{10}^{12}\right)\right)+Y_{1}^{13}\left(1+3\left(1-Y_{10}^{13}\right)\right)+Y_{1}^{14}\left(1+3\left(1-Y_{10}^{14}\right)\right)+Y_{1}^{15}\left(1+3\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_{8}^{1}\times X_{8,1}^{1,2}\Big[Y_{14}^{1}\left(1+3\left(1-Y_{10}^{1}\right)\right)+Y_{14}^{2}\left(1+3\left(1-Y_{10}^{2}\right)\right)+Y_{14}^{3}\left(1+3\left(1-Y_{10}^{3}\right)\right)+$$

$$Y_{14}^{4}\left(1+3\left(1-Y_{10}^{4}\right)\right)+Y_{14}^{5}\left(1+3\left(1-Y_{10}^{5}\right)\right)+Y_{14}^{6}\left(1+3\left(1-Y_{10}^{6}\right)\right)+Y_{14}^{7}\left(1+3\left(1-Y_{10}^{7}\right)\right)+$$

$$Y_{14}^{8}\left(1+3\left(1-Y_{10}^{8}\right)\right)+Y_{14}^{9}\left(1+3\left(1-Y_{10}^{9}\right)\right)+Y_{14}^{10}\left(1+3\left(1-Y_{10}^{10}\right)\right)+Y_{14}^{11}\left(1+3\left(1-Y_{10}^{11}\right)\right)+$$

$$Y_{14}^{12}\left(1+3\left(1-Y_{10}^{12}\right)\right)+Y_{14}^{13}\left(1+3\left(1-Y_{10}^{13}\right)\right)+Y_{14}^{14}\left(1+3\left(1-Y_{10}^{14}\right)\right)+Y_{14}^{15}\left(1+3\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_{8}^{2}\times\Big[Y_{10}^{1}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{1}\right)\right)+Y_{10}^{2}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{2}\right)\right)+Y_{10}^{3}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{3}\right)\right)+$$

$$Y_{10}^{4}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{7}\right)\right)+$$

$$Y_{10}^{8}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{11}\right)\right)+$$

$$Y_{10}^{12}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-X_{8,2}^{2,1}Y_{2}^{15}\right)\right)\Big]$$

$$+10\times Z_{8}^{2}\times\Big[Y_{10}^{1}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{1}\right)\right)+Y_{10}^{2}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{2}\right)\right)+Y_{10}^{3}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{3}\right)\right)+$$

$$Y_{10}^{4}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{4}\right)\right)+Y_{10}^{5}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{5}\right)\right)+Y_{10}^{6}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{6}\right)\right)+Y_{10}^{7}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{7}\right)\right)+$$

$$Y_{10}^{8}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{8}\right)\right)+Y_{10}^{9}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{9}\right)\right)+Y_{10}^{10}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{10}\right)\right)+Y_{10}^{11}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{11}\right)\right)+$$

$$Y_{10}^{12}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{12}\right)\right)+Y_{10}^{13}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{13}\right)\right)+Y_{10}^{14}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{14}\right)\right)+Y_{10}^{15}\left(1+3\left(1-X_{8,15}^{2,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_{8}^{2}\times X_{8,1}^{2,3}\Big[Y_{1}^{1}\left(1+3\left(1-Y_{10}^{1}\right)\right)+Y_{1}^{2}\left(1+3\left(1-Y_{10}^{2}\right)\right)+Y_{1}^{3}\left(1+3\left(1-Y_{10}^{3}\right)\right)+$$

$$Y_{1}^{4}\left(1+3\left(1-Y_{10}^{4}\right)\right)+Y_{1}^{5}\left(1+3\left(1-Y_{10}^{5}\right)\right)+Y_{1}^{6}\left(1+3\left(1-Y_{10}^{6}\right)\right)+Y_{1}^{7}\left(1+3\left(1-Y_{10}^{7}\right)\right)+$$

$$Y_{1}^{8}\left(1+3\left(1-Y_{10}^{8}\right)\right)+Y_{1}^{9}\left(1+3\left(1-Y_{10}^{9}\right)\right)+Y_{1}^{10}\left(1+3\left(1-Y_{10}^{10}\right)\right)+Y_{1}^{11}\left(1+3\left(1-Y_{10}^{11}\right)\right)+$$

$$Y_{1}^{12}\left(1+3\left(1-Y_{10}^{12}\right)\right)+Y_{1}^{13}\left(1+3\left(1-Y_{10}^{13}\right)\right)+Y_{1}^{14}\left(1+3\left(1-Y_{10}^{14}\right)\right)+Y_{1}^{15}\left(1+3\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_{8}^{2}\times X_{8,14}^{2,3}\Big[Y_{14}^{1}\left(1+3\left(1-Y_{10}^{1}\right)\right)+Y_{14}^{2}\left(1+3\left(1-Y_{10}^{2}\right)\right)+Y_{14}^{3}\left(1+3\left(1-Y_{10}^{3}\right)\right)+$$

$$Y_{14}^{4}\left(1+3\left(1-Y_{10}^{4}\right)\right)+Y_{14}^{5}\left(1+3\left(1-Y_{10}^{5}\right)\right)+Y_{14}^{6}\left(1+3\left(1-Y_{10}^{6}\right)\right)+Y_{14}^{7}\left(1+3\left(1-Y_{10}^{7}\right)\right)+$$
$$Y_{14}^{8}\left(1+3\left(1-Y_{10}^{8}\right)\right)+Y_{14}^{9}\left(1+3\left(1-Y_{10}^{9}\right)\right)+Y_{14}^{10}\left(1+3\left(1-Y_{10}^{10}\right)\right)+Y_{14}^{11}\left(1+3\left(1-Y_{10}^{11}\right)\right)+$$
$$Y_{14}^{12}\left(1+3\left(1-Y_{10}^{12}\right)\right)+Y_{14}^{13}\left(1+3\left(1-Y_{10}^{13}\right)\right)+Y_{14}^{14}\left(1+3\left(1-Y_{10}^{14}\right)\right)+Y_{14}^{15}\left(1+3\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_{9}^{1}\times$$

$$\Big[Y_{7}^{1}\left(1+3\left(1-Y_{6}^{1}\right)\right)+Y_{7}^{2}\left(1+3\left(1-Y_{6}^{2}\right)\right)+Y_{7}^{3}\left(1+3\left(1-Y_{6}^{3}\right)\right)+$$
$$Y_{7}^{4}\left(1+3\left(1-Y_{6}^{4}\right)\right)+Y_{7}^{5}\left(1+3\left(1-Y_{6}^{5}\right)\right)+Y_{7}^{6}\left(1+3\left(1-Y_{6}^{6}\right)\right)+Y_{7}^{7}\left(1+3\left(1-Y_{6}^{7}\right)\right)+$$
$$Y_{7}^{8}\left(1+3\left(1-Y_{6}^{8}\right)\right)+Y_{7}^{9}\left(1+3\left(1-Y_{6}^{9}\right)\right)+Y_{7}^{10}\left(1+3\left(1-Y_{6}^{10}\right)\right)+Y_{7}^{11}\left(1+3\left(1-Y_{6}^{11}\right)\right)+$$
$$Y_{7}^{12}\left(1+3\left(1-Y_{6}^{12}\right)\right)+Y_{7}^{13}\left(1+3\left(1-Y_{6}^{13}\right)\right)+Y_{7}^{14}\left(1+3\left(1-Y_{6}^{14}\right)\right)+Y_{7}^{15}\left(1+3\left(1-Y_{6}^{15}\right)\right)\Big]$$

$$+10\times Z_{9}^{1}\times$$

$$\Big[Y_{8}^{1}\left(1+3\left(1-Y_{7}^{1}\right)\right)+Y_{8}^{2}\left(1+3\left(1-Y_{7}^{2}\right)\right)+Y_{8}^{3}\left(1+3\left(1-Y_{7}^{3}\right)\right)+$$
$$Y_{8}^{4}\left(1+3\left(1-Y_{7}^{4}\right)\right)+Y_{8}^{5}\left(1+3\left(1-Y_{7}^{5}\right)\right)+Y_{8}^{6}\left(1+3\left(1-Y_{7}^{6}\right)\right)+Y_{8}^{7}\left(1+3\left(1-Y_{7}^{7}\right)\right)+$$
$$Y_{8}^{8}\left(1+3\left(1-Y_{7}^{8}\right)\right)+Y_{8}^{9}\left(1+3\left(1-Y_{7}^{9}\right)\right)+Y_{8}^{10}\left(1+3\left(1-Y_{7}^{10}\right)\right)+Y_{8}^{11}\left(1+3\left(1-Y_{7}^{11}\right)\right)+$$
$$Y_{8}^{12}\left(1+3\left(1-Y_{7}^{12}\right)\right)+Y_{8}^{13}\left(1+3\left(1-Y_{7}^{13}\right)\right)+Y_{8}^{14}\left(1+3\left(1-Y_{7}^{14}\right)\right)+Y_{8}^{15}\left(1+3\left(1-Y_{7}^{15}\right)\right)\Big]$$

$$+10\times Z_{9}^{2}\times\Big[Y_{6}^{1}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{1}\right)\right)+Y_{6}^{2}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{2}\right)\right)+Y_{6}^{3}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{3}\right)\right)+$$
$$Y_{6}^{4}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{4}\right)\right)+Y_{6}^{5}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{5}\right)\right)+Y_{6}^{6}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{6}\right)\right)+Y_{6}^{7}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{7}\right)\right)+$$
$$Y_{6}^{8}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{8}\right)\right)+Y_{6}^{9}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{9}\right)\right)+Y_{6}^{10}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{10}\right)\right)+Y_{6}^{11}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{11}\right)\right)+$$
$$Y_{6}^{12}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{12}\right)\right)+Y_{6}^{13}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{13}\right)\right)+Y_{6}^{14}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{14}\right)\right)+Y_{6}^{15}\left(1+3\left(1-X_{9,1}^{2,1}Y_{1}^{15}\right)\right)\Big]$$

$$+10\times Z_{9}^{2}\times\Big[Y_{6}^{1}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{1}\right)\right)+Y_{6}^{2}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{2}\right)\right)+Y_{6}^{3}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{3}\right)\right)+$$
$$Y_{6}^{4}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{4}\right)\right)+Y_{6}^{5}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{5}\right)\right)+Y_{6}^{6}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{6}\right)\right)+Y_{6}^{7}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{7}\right)\right)+$$
$$Y_{6}^{8}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{8}\right)\right)+Y_{6}^{9}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{9}\right)\right)+Y_{6}^{10}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{10}\right)\right)+Y_{6}^{11}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{11}\right)\right)+$$
$$Y_{6}^{12}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{12}\right)\right)+Y_{6}^{13}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{13}\right)\right)+Y_{6}^{14}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{14}\right)\right)+Y_{6}^{15}\left(1+3\left(1-X_{9,14}^{2,1}Y_{14}^{15}\right)\right)\Big]$$

$$+10\times Z_{9}^{2}\times\Big[Y_{8}^{1}\left(1+3\left(1-Y_{6}^{1}\right)\right)+Y_{8}^{2}\left(1+3\left(1-Y_{6}^{2}\right)\right)+Y_{8}^{3}\left(1+3\left(1-Y_{6}^{3}\right)\right)+$$
$$Y_{8}^{4}\left(1+3\left(1-Y_{6}^{4}\right)\right)+Y_{8}^{5}\left(1+3\left(1-Y_{6}^{5}\right)\right)+Y_{8}^{6}\left(1+3\left(1-Y_{6}^{6}\right)\right)+Y_{8}^{7}\left(1+3\left(1-Y_{6}^{7}\right)\right)+$$
$$Y_{8}^{8}\left(1+3\left(1-Y_{6}^{8}\right)\right)+Y_{8}^{9}\left(1+3\left(1-Y_{6}^{9}\right)\right)+Y_{8}^{10}\left(1+3\left(1-Y_{6}^{10}\right)\right)+Y_{8}^{11}\left(1+3\left(1-Y_{6}^{11}\right)\right)+$$
$$Y_{8}^{12}\left(1+3\left(1-Y_{6}^{12}\right)\right)+Y_{8}^{13}\left(1+3\left(1-Y_{6}^{13}\right)\right)+Y_{8}^{14}\left(1+3\left(1-Y_{6}^{14}\right)\right)+Y_{8}^{15}\left(1+3\left(1-Y_{6}^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^{1}\times\Big[Y_{4}^{1}\left(1+3\left(1-Y_{9}^{1}\right)\right)+Y_{4}^{2}\left(1+3\left(1-Y_{9}^{2}\right)\right)+Y_{4}^{3}\left(1+3\left(1-Y_{9}^{3}\right)\right)+$$

$$Y_4^4\left(1+3\left(1-Y_9^4\right)\right)+Y_4^5\left(1+3\left(1-Y_9^5\right)\right)+Y_4^6\left(1+3\left(1-Y_9^6\right)\right)+Y_4^7\left(1+3\left(1-Y_9^7\right)\right)+$$
$$Y_4^8\left(1+3\left(1-Y_9^8\right)\right)+Y_4^9\left(1+3\left(1-Y_9^9\right)\right)+Y_4^{10}\left(1+3\left(1-Y_9^{10}\right)\right)+Y_4^{11}\left(1+3\left(1-Y_9^{11}\right)\right)+$$
$$Y_4^{12}\left(1+3\left(1-Y_9^{12}\right)\right)+Y_4^{13}\left(1+3\left(1-Y_9^{13}\right)\right)+Y_4^{14}\left(1+3\left(1-Y_9^{14}\right)\right)+Y_4^{15}\left(1+3\left(1-Y_9^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^1\times X_{10,1}^{1,2}\Big[Y_1^1\left(1+3\left(1-Y_4^1\right)\right)+Y_1^2\left(1+3\left(1-Y_4^2\right)\right)+Y_1^3\left(1+3\left(1-Y_4^3\right)\right)+$$
$$Y_1^4\left(1+3\left(1-Y_4^4\right)\right)+Y_1^5\left(1+3\left(1-Y_4^5\right)\right)+Y_1^6\left(1+3\left(1-Y_4^6\right)\right)+Y_1^7\left(1+3\left(1-Y_4^7\right)\right)+$$
$$Y_1^8\left(1+3\left(1-Y_4^8\right)\right)+Y_1^9\left(1+3\left(1-Y_4^9\right)\right)+Y_1^{10}\left(1+3\left(1-Y_4^{10}\right)\right)+Y_1^{11}\left(1+3\left(1-Y_4^{11}\right)\right)+$$
$$Y_1^{12}\left(1+3\left(1-Y_4^{12}\right)\right)+Y_1^{13}\left(1+3\left(1-Y_4^{13}\right)\right)+Y_1^{14}\left(1+3\left(1-Y_4^{14}\right)\right)+Y_1^{15}\left(1+3\left(1-Y_4^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^1\times X_{10,14}^{1,2}\Big[Y_{14}^1\left(1+3\left(1-Y_4^1\right)\right)+Y_{14}^2\left(1+3\left(1-Y_4^2\right)\right)+Y_{14}^3\left(1+3\left(1-Y_4^3\right)\right)+$$
$$Y_{14}^4\left(1+3\left(1-Y_4^4\right)\right)+Y_{14}^5\left(1+3\left(1-Y_4^5\right)\right)+Y_{14}^6\left(1+3\left(1-Y_4^6\right)\right)+Y_{14}^7\left(1+3\left(1-Y_4^7\right)\right)+$$
$$Y_{14}^8\left(1+3\left(1-Y_4^8\right)\right)+Y_{14}^9\left(1+3\left(1-Y_4^9\right)\right)+Y_{14}^{10}\left(1+3\left(1-Y_4^{10}\right)\right)+Y_{14}^{11}\left(1+3\left(1-Y_4^{11}\right)\right)+$$
$$Y_{14}^{12}\left(1+3\left(1-Y_4^{12}\right)\right)+Y_{14}^{13}\left(1+3\left(1-Y_4^{13}\right)\right)+Y_{14}^{14}\left(1+3\left(1-Y_4^{14}\right)\right)+Y_{14}^{15}\left(1+3\left(1-Y_4^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^2\times$$

$$\Big[Y_6^1\left(1+3\left(1-Y_8^1\right)\right)+Y_6^2\left(1+3\left(1-Y_8^2\right)\right)+Y_6^3\left(1+3\left(1-Y_8^3\right)\right)+Y_6^4\left(1+3\left(1-Y_8^4\right)\right)$$
$$+Y_6^5\left(1+3\left(1-Y_8^{51}\right)\right)+Y_6^6\left(1+3\left(1-Y_8^6\right)\right)+Y_6^7\left(1+3\left(1-Y_8^7\right)\right)+Y_6^8\left(1+3\left(1-Y_8^{81}\right)\right)$$
$$+Y_6^9\left(1+3\left(1-Y_8^{91}\right)\right)+Y_6^{10}\left(1+3\left(1-Y_8^{10}\right)\right)+Y_6^{11}\left(1+3\left(1-Y_8^{11}\right)\right)+Y_6^{12}\left(1+3\left(1-Y_8^{12}\right)\right)$$
$$+Y_6^{13}\left(1+3\left(1-Y_8^{13}\right)\right)+Y_6^{14}\left(1+3\left(1-Y_8^{14}\right)\right)+Y_6^{15}\left(1+3\left(1-Y_8^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^2\times$$

$$\Big[Y_5^1\left(1+3\left(1-Y_6^1\right)\right)+Y_5^2\left(1+3\left(1-Y_6^2\right)\right)+Y_5^3\left(1+3\left(1-Y_6^3\right)\right)+Y_5^4\left(1+3\left(1-Y_6^4\right)\right)$$
$$+Y_5^5\left(1+3\left(1-Y_6^5\right)\right)+Y_5^6\left(1+3\left(1-Y_6^6\right)\right)+Y_5^7\left(1+3\left(1-Y_6^7\right)\right)+Y_5^8\left(1+3\left(1-Y_6^8\right)\right)$$
$$+Y_5^9\left(1+3\left(1-Y_6^9\right)\right)+Y_5^{10}\left(1+3\left(1-Y_6^{10}\right)\right)+Y_5^{11}\left(1+3\left(1-Y_6^{11}\right)\right)+Y_5^{12}\left(1+3\left(1-Y_6^{12}\right)\right)$$
$$+Y_5^{13}\left(1+3\left(1-Y_6^{13}\right)\right)+Y_5^{14}\left(1+3\left(1-Y_6^{14}\right)\right)+Y_5^{15}\left(1+3\left(1-Y_6^{15}\right)\right)\Big]$$

A-2.1b *Model 2 :*

To minimize

$$10\left[Z_1^1\{(5\times 5\times 1)+(4\times 4\times 1)+(3\times 3\times 3)\}+Z_1^2\{(5\times 5\times 1)+(4\times 4\times 1)+(3\times 3\times 3)\}\right]+$$

$$10\left[Z_2^1\{(5\times 5\times 1)+(4\times 4\times 1)+(3\times 3\times 3)\}+Z_2^2\{(5\times 5\times 1)+(4\times 4\times 1)+(3\times 3\times 3)\}\right]+$$

$$10\left[Z_3^1\{(5\times5\times1)+(4\times4\times1)\}+Z_3^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_4^1\{(5\times5\times1)+(4\times4\times1)\}+Z_4^2\{(5\times5\times1)+(4\times4\times1)\}\right]+$$

$$10\left[Z_5^1\{(5\times5\times1)+(4\times4\times1)\}+Z_5^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_6^1\{(5\times5\times1)+(4\times4\times1)\}+Z_6^2\{(5\times5\times1)+(4\times4\times1)\}\right]+$$

$$10\left[Z_7^1\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}+Z_7^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_8^1\{(5\times5\times1)+(4\times4\times1)\}+Z_8^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_9^1\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}+Z_9^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\left[Z_{10}^1\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}+Z_{10}^2\{(5\times5\times1)+(4\times4\times1)+(3\times3\times3)\}\right]+$$

$$10\times Z_1^1\times4\times4\times\left[Y_3^1\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^1\right)\right)+Y_3^2\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^2\right)\right)+Y_3^3\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^3\right)\right)+\right.$$

$$Y_3^4\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^4\right)\right)+Y_3^5\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^5\right)\right)+Y_3^6\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^6\right)\right)+Y_3^7\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^7\right)\right)+$$

$$Y_3^8\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^8\right)\right)+Y_3^9\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^9\right)\right)+Y_3^{10}\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^{10}\right)\right)+Y_3^{11}\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^{11}\right)\right)+$$

$$\left.Y_3^{12}\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^{12}\right)\right)+Y_3^{13}\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^{13}\right)\right)+Y_3^{14}\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^{14}\right)\right)+Y_3^{15}\left(0.2+0.8\left(1-X_{1,1}^{1,1}Y_1^{15}\right)\right)\right]$$

$$+10\times Z_1^1\times4\times4\times\left[Y_3^1\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^1\right)\right)+Y_3^2\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^2\right)\right)+Y_3^3\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^3\right)\right)+\right.$$

$$Y_3^4\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^4\right)\right)+Y_3^5\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^5\right)\right)+Y_3^6\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^6\right)\right)+Y_3^7\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^7\right)\right)+$$

$$Y_3^8\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^8\right)\right)+Y_3^9\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^9\right)\right)+Y_3^{10}\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^{10}\right)\right)+Y_3^{11}\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^{11}\right)\right)+$$

$$\left.Y_3^{12}\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^{12}\right)\right)+Y_3^{13}\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^{13}\right)\right)+Y_3^{14}\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^{14}\right)\right)+Y_3^{15}\left(0.2+0.8\left(1-X_{1,14}^{1,1}Y_{14}^{15}\right)\right)\right]$$

$$+10\times Z_1^1\times3\times3\times\left[Y_{10}^1\left(0.2+0.8\left(1-Y_3^1\right)\right)+Y_{10}^2\left(0.2+0.8\left(1-Y_3^2\right)\right)+Y_{10}^3\left(0.2+0.8\left(1-Y_3^3\right)\right)+\right.$$

$$Y_{10}^4\left(0.2+0.8\left(1-Y_3^4\right)\right)+Y_{10}^5\left(0.2+0.8\left(1-Y_3^5\right)\right)+Y_{10}^6\left(0.2+0.8\left(1-Y_3^6\right)\right)+Y_{10}^7\left(0.2+0.8\left(1-Y_3^7\right)\right)+$$

$$Y_{10}^8\left(0.2+0.8\left(1-Y_3^8\right)\right)+Y_{10}^9\left(0.2+0.8\left(1-Y_3^9\right)\right)+Y_{10}^{10}\left(0.2+0.8\left(1-Y_3^{10}\right)\right)+Y_{10}^{11}\left(0.2+0.8\left(1-Y_3^{11}\right)\right)+$$

$$\left.Y_{10}^{12}\left(0.2+0.8\left(1-Y_3^{12}\right)\right)+Y_{10}^{13}\left(0.2+0.8\left(1-Y_3^{13}\right)\right)+Y_{10}^{14}\left(0.2+0.8\left(1-Y_3^{14}\right)\right)+Y_{10}^{15}\left(0.2+0.8\left(1-Y_3^{15}\right)\right)\right]$$

$$+10\times Z_1^2\times4\times4\times\left[Y_3^1\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^1\right)\right)+Y_3^2\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^2\right)\right)+Y_3^3\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^3\right)\right)+\right.$$

$$Y_3^4\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^4\right)\right)+Y_3^5\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^5\right)\right)+Y_3^6\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^6\right)\right)+Y_3^7\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^7\right)\right)+$$

$$Y_3^8\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^8\right)\right)+Y_3^9\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^9\right)\right)+Y_3^{10}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{10}\right)\right)+Y_3^{11}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{11}\right)\right)+$$

$$\left.Y_3^{12}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{12}\right)\right)+Y_3^{13}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{13}\right)\right)+Y_3^{14}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{14}\right)\right)+Y_3^{15}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{15}\right)\right)\right]$$

$$+10\times Z_1^2\times4\times4\times\left[Y_3^1\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^1\right)\right)+Y_3^2\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^2\right)\right)+Y_3^3\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^3\right)\right)+\right.$$

$$Y_3^4\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^4\right)\right)+Y_3^5\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^5\right)\right)+Y_3^6\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^6\right)\right)+Y_3^7\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^7\right)\right)+$$

$$Y_3^8\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^8\right)\right)+Y_3^9\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^9\right)\right)+Y_3^{10}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{10}\right)\right)+Y_3^{11}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{11}\right)\right)+$$

$$Y_3^{12}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{12}\right)\right)+Y_3^{13}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{13}\right)\right)+Y_3^{14}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{14}\right)\right)+Y_3^{15}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_1^2\times 3\times 3\times\Big[Y_{13}^1\left(0.2+0.8\left(1-Y_3^1\right)\right)+Y_{13}^2\left(0.2+0.8\left(1-Y_3^2\right)\right)+Y_{13}^3\left(0.2+0.8\left(1-Y_3^3\right)\right)+$$

$$Y_{13}^4\left(0.2+0.8\left(1-Y_3^4\right)\right)+Y_{13}^5\left(0.2+0.8\left(1-Y_3^5\right)\right)+Y_{13}^6\left(0.2+0.8\left(1-Y_3^6\right)\right)+Y_{13}^7\left(0.2+0.8\left(1-Y_3^7\right)\right)+$$

$$Y_{13}^8\left(0.2+0.8\left(1-Y_3^8\right)\right)+Y_{13}^9\left(0.2+0.8\left(1-Y_3^9\right)\right)+Y_{13}^{10}\left(0.2+0.8\left(1-Y_3^{10}\right)\right)+Y_{13}^{11}\left(0.2+0.8\left(1-Y_3^{11}\right)\right)+$$

$$Y_{13}^{12}\left(0.2+0.8\left(1-Y_3^{12}\right)\right)+Y_{13}^{13}\left(0.2+0.8\left(1-Y_3^{13}\right)\right)+Y_{13}^{14}\left(0.2+0.8\left(1-Y_3^{14}\right)\right)+Y_{13}^{15}\left(0.2+0.8\left(1-Y_3^{15}\right)\right)\Big]$$

$$+10\times Z_2^1\times 4\times 4\times\Big[Y_8^1\left(0.2+0.8\left(1-Y_{10}^1\right)\right)+Y_8^2\left(0.2+0.8\left(1-Y_{10}^2\right)\right)+Y_8^3\left(0.2+0.8\left(1-Y_{10}^3\right)\right)+$$

$$Y_8^4\left(0.2+0.8\left(1-Y_{10}^4\right)\right)+Y_8^5\left(0.2+0.8\left(1-Y_{10}^5\right)\right)+Y_8^6\left(0.2+0.8\left(1-Y_{10}^6\right)\right)+Y_8^7\left(0.2+0.8\left(1-Y_{10}^7\right)\right)+$$

$$Y_8^8\left(0.2+0.8\left(1-Y_{10}^8\right)\right)+Y_8^9\left(0.2+0.8\left(1-Y_{10}^9\right)\right)+Y_8^{10}\left(0.2+0.8\left(1-Y_{10}^{10}\right)\right)+Y_8^{11}\left(0.2+0.8\left(1-Y_{10}^{11}\right)\right)+$$

$$Y_8^{12}\left(0.2+0.8\left(1-Y_{10}^{12}\right)\right)+Y_8^{13}\left(0.2+0.8\left(1-Y_{10}^{13}\right)\right)+Y_8^{14}\left(0.2+0.8\left(1-Y_{10}^{14}\right)\right)+Y_8^{15}\left(0.2+0.8\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_2^2\times 4\times 4\times X_{2,1}^{2,2}\Big[Y_1^1\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^1\right)\right)+Y_1^2\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^2\right)\right)+Y_1^3\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^3\right)\right)+$$

$$Y_1^4\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^4\right)\right)+Y_1^5\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^5\right)\right)+Y_1^6\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^6\right)\right)+Y_1^7\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^7\right)\right)+$$

$$Y_1^8\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^8\right)\right)+Y_1^9\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^9\right)\right)+Y_1^{10}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{10}\right)\right)+Y_1^{11}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{11}\right)\right)+$$

$$Y_1^{12}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{12}\right)\right)+Y_1^{13}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{13}\right)\right)+Y_1^{14}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{14}\right)\right)+Y_1^{15}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{15}\right)\right)\Big]$$

$$+10\times Z_2^2\times 4\times 4\times X_{2,14}^{2,2}$$

$$\Big[Y_{14}^1\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^1\right)\right)+Y_{14}^2\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^2\right)\right)+Y_{14}^3\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^3\right)\right)+$$

$$Y_{14}^4\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^4\right)\right)+Y_{14}^5\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^5\right)\right)+Y_{14}^6\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^6\right)\right)+Y_{14}^7\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^7\right)\right)+$$

$$Y_{14}^8\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^8\right)\right)+Y_{14}^9\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^9\right)\right)+Y_{14}^{10}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{10}\right)\right)+Y_{14}^{11}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{11}\right)\right)+$$

$$Y_{14}^{12}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{12}\right)\right)+Y_{14}^{13}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{13}\right)\right)+Y_{14}^{14}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{14}\right)\right)+Y_{14}^{15}\left(0.2+0.8\left(1-X_{1,2}^{2,1}Y_2^{15}\right)\right)\Big]$$

$$+10\times Z_2^2\times 4\times 4\times X_{2,1}^{2,2}$$

$$\Big[Y_1^1\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^1\right)\right)+Y_1^2\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^2\right)\right)+Y_1^3\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^3\right)\right)+$$

$$Y_1^4\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^4\right)\right)+Y_1^5\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^5\right)\right)+Y_1^6\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^6\right)\right)+Y_1^7\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^7\right)\right)+$$

$$Y_1^8\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^8\right)\right)+Y_1^9\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^9\right)\right)+Y_1^{10}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{10}\right)\right)+Y_1^{11}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{11}\right)\right)+$$

$$Y_1^{12}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{12}\right)\right)+Y_1^{13}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{13}\right)\right)+Y_1^{14}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{14}\right)\right)+Y_1^{15}\left(0.2+0.8\left(1-X_{1,15}^{2,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_2^2\times 4\times 4\times X_{2,14}^{2,2}$$

$$\Big[Y_{14}^1\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^1\big)\big)+Y_{14}^2\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^2\big)\big)+Y_{14}^3\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^3\big)\big)+$$

$$Y_{14}^4\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^4\big)\big)+Y_{14}^5\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^5\big)\big)+Y_{14}^6\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^6\big)\big)+Y_{14}^7\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^7\big)\big)+$$

$$Y_{14}^8\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^8\big)\big)+Y_{14}^9\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^9\big)\big)+Y_{14}^{10}\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^{10}\big)\big)+Y_{14}^{11}\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^{11}\big)\big)+$$

$$Y_{14}^{12}\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^{12}\big)\big)+Y_{14}^{13}\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^{13}\big)\big)+Y_{14}^{14}\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^{14}\big)\big)+Y_{14}^{15}\big(0.2+0.8\big(1-X_{1,15}^{2,1}Y_{15}^{15}\big)\big)\Big]$$

$$+10\times Z_2^2\times 3\times 3\times\Big[Y_{10}^1\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^1\big)\big)+Y_{10}^2\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^2\big)\big)+Y_{10}^3\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^3\big)\big)+$$

$$Y_{10}^4\big(0.2+0.8\big(1-X_{2,1}^{2,2}X_{1,1}^{1,1}Y_1^4\big)\big)+Y_{10}^5\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^5\big)\big)+Y_{10}^6\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^6\big)\big)+Y_{10}^7\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^7\big)\big)+$$

$$Y_{10}^8\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^8\big)\big)+Y_{10}^9\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^9\big)\big)+Y_{10}^{10}\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^{10}\big)\big)+Y_{10}^{11}\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^{11}\big)\big)+$$

$$Y_{10}^{12}\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^{12}\big)\big)+Y_{10}^{13}\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^{13}\big)\big)+Y_{10}^{14}\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^{14}\big)\big)+Y_{10}^{15}\big(0.2+0.8\big(1-X_{2,1}^{2,2}Y_1^{15}\big)\big)\Big]$$

$$+10\times Z_2^2\times 3\times 3\times\Big[Y_{10}^1\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^1\big)\big)+Y_{10}^2\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^2\big)\big)+Y_{10}^3\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^3\big)\big)+$$

$$Y_{10}^4\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^4\big)\big)+Y_{10}^5\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^5\big)\big)+Y_{10}^6\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^6\big)\big)+Y_{10}^7\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^7\big)\big)+$$

$$Y_{10}^8\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^8\big)\big)+Y_{10}^9\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^9\big)\big)+Y_{10}^{10}\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^{10}\big)\big)+Y_{10}^{11}\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^{11}\big)\big)+$$

$$Y_{10}^{12}\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^{12}\big)\big)+Y_{10}^{13}\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^{13}\big)\big)+Y_{10}^{14}\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^{14}\big)\big)+Y_{10}^{15}\big(0.2+0.8\big(1-X_{2,14}^{2,2}Y_{14}^{15}\big)\big)\Big]$$

$$+10\times Z_3^1\times 4\times 4\times\Big[Y_{10}^1\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^1\big)\big)+Y_{10}^2\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^2\big)\big)+Y_{10}^3\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^3\big)\big)+$$

$$Y_{10}^4\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^4\big)\big)+Y_{10}^5\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^5\big)\big)+Y_{10}^6\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^6\big)\big)+Y_{10}^7\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^7\big)\big)+$$

$$Y_{10}^8\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^8\big)\big)+Y_{10}^9\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^9\big)\big)+Y_{10}^{10}\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^{10}\big)\big)+Y_{10}^{11}\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^{11}\big)\big)+$$

$$Y_{10}^{12}\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^{12}\big)\big)+Y_{10}^{13}\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^{13}\big)\big)+Y_{10}^{14}\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^{14}\big)\big)+Y_{10}^{15}\big(0.2+0.8\big(1-X_{3,2}^{1,1}Y_2^{15}\big)\big)\Big]$$

$$+10\times Z_3^1\times 4\times 4\times\Big[Y_{10}^1\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^1\big)\big)+Y_{10}^2\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^2\big)\big)+Y_{10}^3\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^3\big)\big)+$$

$$Y_{10}^4\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^4\big)\big)+Y_{10}^5\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^5\big)\big)+Y_{10}^6\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^6\big)\big)+Y_{10}^7\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^7\big)\big)+$$

$$Y_{10}^8\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^8\big)\big)+Y_{10}^9\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^9\big)\big)+Y_{10}^{10}\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^{10}\big)\big)+Y_{10}^{11}\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^{11}\big)\big)+$$

$$Y_{10}^{12}\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^{12}\big)\big)+Y_{10}^{13}\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^{13}\big)\big)+Y_{10}^{14}\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^{14}\big)\big)+Y_{10}^{15}\big(0.2+0.8\big(1-X_{3,15}^{1,1}Y_{15}^{15}\big)\big)\Big]$$

$$+10\times Z_3^2\times 4\times 4\times\Big[Y_{11}^1\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^1\big)\big)+Y_{11}^2\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^2\big)\big)+Y_{11}^3\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^3\big)\big)+$$

$$Y_{11}^4\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^4\big)\big)+Y_{11}^5\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^5\big)\big)+Y_{11}^6\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^6\big)\big)+Y_{11}^7\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^7\big)\big)+$$

$$Y_{11}^8\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^8\big)\big)+Y_{11}^9\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^9\big)\big)+Y_{11}^{10}\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^{10}\big)\big)+Y_{11}^{11}\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^{11}\big)\big)+$$

$$Y_{11}^{12}\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^{12}\big)\big)+Y_{11}^{13}\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^{13}\big)\big)+Y_{11}^{14}\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^{14}\big)\big)+Y_{11}^{15}\big(0.2+0.8\big(1-X_{3,1}^{2,1}Y_1^{15}\big)\big)\Big]$$

$$+10\times Z_3^2\times 4\times 4\times\Big[Y_{11}^1\big(0.2+0.8\big(1-X_{3,14}^{2,1}Y_{14}^1\big)\big)+Y_{11}^2\big(0.2+0.8\big(1-X_{3,14}^{2,1}Y_{14}^2\big)\big)+Y_{11}^3\big(0.2+0.8\big(1-X_{3,14}^{2,1}Y_{14}^3\big)\big)+$$

$$Y_{11}^{4}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{4}\right)\right)+Y_{11}^{5}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{5}\right)\right)+Y_{11}^{6}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{6}\right)\right)+Y_{11}^{7}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{7}\right)\right)+$$

$$Y_{11}^{8}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{8}\right)\right)+Y_{11}^{9}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{9}\right)\right)+Y_{11}^{10}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{10}\right)\right)+Y_{11}^{11}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{11}\right)\right)+$$

$$Y_{11}^{12}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{12}\right)\right)+Y_{11}^{13}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{13}\right)\right)+Y_{11}^{14}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{14}\right)\right)+Y_{11}^{15}\left(0.2+0.8\left(1-X_{3,14}^{2,1}Y_{14}^{15}\right)\right)\Big]$$

$$+10\times Z_{4}^{1}\times 4\times 4\times X_{4,1}^{1,2}\Big[Y_{1}^{1}\left(0.2+0.8\left(1-Y_{3}^{1}\right)\right)+Y_{1}^{2}\left(0.2+0.8\left(1-Y_{3}^{2}\right)\right)+Y_{1}^{3}\left(0.2+0.8\left(1-Y_{3}^{3}\right)\right)+$$

$$Y_{1}^{4}\left(0.2+0.8\left(1-Y_{3}^{4}\right)\right)+Y_{1}^{5}\left(0.2+0.8\left(1-Y_{3}^{5}\right)\right)+Y_{1}^{6}\left(0.2+0.8\left(1-Y_{3}^{6}\right)\right)+Y_{1}^{7}\left(0.2+0.8\left(1-Y_{3}^{7}\right)\right)+$$

$$Y_{1}^{8}\left(0.2+0.8\left(1-Y_{3}^{8}\right)\right)+Y_{1}^{9}\left(0.2+0.8\left(1-Y_{3}^{9}\right)\right)+Y_{1}^{10}\left(0.2+0.8\left(1-Y_{3}^{10}\right)\right)+Y_{1}^{11}\left(0.2+0.8\left(1-Y_{3}^{11}\right)\right)+$$

$$Y_{1}^{12}\left(0.2+0.8\left(1-Y_{3}^{12}\right)\right)+Y_{1}^{13}\left(0.2+0.8\left(1-Y_{3}^{13}\right)\right)+Y_{1}^{14}\left(0.2+0.8\left(1-Y_{3}^{14}\right)\right)+Y_{1}^{15}\left(0.2+0.8\left(1-Y_{3}^{15}\right)\right)\Big]$$

$$+10\times Z_{4}^{1}\times 4\times 4\times X_{4,14}^{1,2}\Big[Y_{14}^{1}\left(0.2+0.8\left(1-Y_{3}^{1}\right)\right)+Y_{14}^{2}\left(0.2+0.8\left(1-Y_{3}^{2}\right)\right)+Y_{14}^{3}\left(0.2+0.8\left(1-Y_{3}^{3}\right)\right)+$$

$$Y_{14}^{4}\left(0.2+0.8\left(1-Y_{3}^{4}\right)\right)+Y_{14}^{5}\left(0.2+0.8\left(1-Y_{3}^{5}\right)\right)+Y_{14}^{6}\left(0.2+0.8\left(1-Y_{3}^{6}\right)\right)+Y_{14}^{7}\left(0.2+0.8\left(1-Y_{3}^{7}\right)\right)+$$

$$Y_{14}^{8}\left(0.2+0.8\left(1-Y_{3}^{8}\right)\right)+Y_{14}^{9}\left(0.2+0.8\left(1-Y_{3}^{9}\right)\right)+Y_{14}^{10}\left(0.2+0.8\left(1-Y_{3}^{10}\right)\right)+Y_{14}^{11}\left(0.2+0.8\left(1-Y_{3}^{11}\right)\right)+$$

$$Y_{14}^{12}\left(0.2+0.8\left(1-Y_{3}^{12}\right)\right)+Y_{14}^{13}\left(0.2+0.8\left(1-Y_{3}^{13}\right)\right)+Y_{14}^{14}\left(0.2+0.8\left(1-Y_{3}^{14}\right)\right)+Y_{14}^{15}\left(0.2+0.8\left(1-Y_{3}^{15}\right)\right)\Big]$$

$$+10\times Z_{4}^{2}\times 4\times 4\times X_{4,2}^{2,2}\Big[Y_{2}^{1}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{1}\right)\right)+Y_{2}^{2}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{2}\right)\right)+Y_{2}^{3}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{3}\right)\right)+$$

$$Y_{2}^{4}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{4}\right)\right)+Y_{2}^{5}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{5}\right)\right)+Y_{2}^{6}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{6}\right)\right)+Y_{2}^{7}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{7}\right)\right)+$$

$$Y_{2}^{8}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{8}\right)\right)+Y_{2}^{9}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{9}\right)\right)+Y_{2}^{10}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{10}\right)\right)+Y_{2}^{11}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{11}\right)\right)+$$

$$Y_{2}^{12}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{12}\right)\right)+Y_{2}^{13}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{13}\right)\right)+Y_{2}^{14}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{14}\right)\right)+Y_{2}^{15}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{15}\right)\right)\Big]$$

$$+10\times Z_{4}^{2}\times 4\times 4\times X_{4,15}^{2,2}$$

$$\Big[Y_{15}^{1}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{1}\right)\right)+Y_{15}^{2}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{2}\right)\right)+Y_{15}^{3}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{3}\right)\right)+$$

$$Y_{15}^{4}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{4}\right)\right)+Y_{15}^{5}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{5}\right)\right)+Y_{15}^{6}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{6}\right)\right)+Y_{15}^{7}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{7}\right)\right)+$$

$$Y_{15}^{8}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{8}\right)\right)+Y_{15}^{9}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{9}\right)\right)+Y_{15}^{10}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{10}\right)\right)+Y_{15}^{11}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{11}\right)\right)+$$

$$Y_{15}^{12}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{12}\right)\right)+Y_{15}^{13}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{13}\right)\right)+Y_{15}^{14}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{14}\right)\right)+Y_{15}^{15}\left(0.2+0.8\left(1-X_{4,1}^{2,1}Y_{1}^{15}\right)\right)\Big]$$

$$+10\times Z_{4}^{2}\times 4\times 4\times X_{4,2}^{2,2}$$

$$\Big[Y_{2}^{1}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{1}\right)\right)+Y_{2}^{2}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{2}\right)\right)+Y_{2}^{3}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{3}\right)\right)+$$

$$Y_{2}^{4}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{4}\right)\right)+Y_{2}^{5}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{5}\right)\right)+Y_{2}^{6}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{6}\right)\right)+Y_{2}^{7}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{7}\right)\right)+$$

$$Y_{2}^{8}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{8}\right)\right)+Y_{2}^{9}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{9}\right)\right)+Y_{2}^{10}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{10}\right)\right)+Y_{2}^{11}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{11}\right)\right)+$$

$$Y_{2}^{12}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{12}\right)\right)+Y_{2}^{13}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{13}\right)\right)+Y_{2}^{14}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{14}\right)\right)+Y_{2}^{15}\left(0.2+0.8\left(1-X_{4,14}^{2,1}Y_{14}^{15}\right)\right)\Big]$$

$$+10\times Z_4^2\times 4\times 4\times X_{4,15}^{2,2}$$

$$\Big[Y_{15}^1\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^1\big)\big)+Y_{15}^2\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^2\big)\big)+Y_{15}^3\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^3\big)\big)+$$

$$Y_{15}^4\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^4\big)\big)+Y_{15}^5\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^5\big)\big)+Y_{15}^6\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^6\big)\big)+Y_{15}^7\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^7\big)\big)+$$

$$Y_{15}^8\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^8\big)\big)+Y_{15}^9\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^9\big)\big)+Y_{15}^{10}\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^{10}\big)\big)+Y_{15}^{11}\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^{11}\big)\big)+$$

$$Y_{15}^{12}\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^{12}\big)\big)+Y_{15}^{13}\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^{13}\big)\big)+Y_{15}^{14}\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^{14}\big)\big)+Y_{15}^{15}\big(0.2+0.8\big(1-X_{4,14}^{2,1}Y_{14}^{15}\big)\big)\Big]$$

$$+10\times Z_5^1\times 4\times 4$$

$$\Big[Y_{11}^1\big(0.2+0.8\big(1-Y_3^1\big)\big)+Y_{11}^2\big(0.2+0.8\big(1-Y_3^2\big)\big)+Y_{11}^3\big(0.2+0.8\big(1-Y_3^3\big)\big)+$$

$$Y_{11}^4\big(0.2+0.8\big(1-Y_3^4\big)\big)+Y_{11}^5\big(0.2+0.8\big(1-Y_3^5\big)\big)+Y_{11}^6\big(0.2+0.8\big(1-Y_3^6\big)\big)+Y_{11}^7\big(0.2+0.8\big(1-Y_3^7\big)\big)+$$

$$Y_{11}^8\big(0.2+0.8\big(1-Y_3^8\big)\big)+Y_{11}^9\big(0.2+0.8\big(1-Y_3^9\big)\big)+Y_{11}^{10}\big(0.2+0.8\big(1-Y_3^{10}\big)\big)+Y_{11}^{11}\big(0.2+0.8\big(1-Y_3^{11}\big)\big)+$$

$$Y_{11}^{12}\big(0.2+0.8\big(1-Y_3^{12}\big)\big)+Y_{11}^{13}\big(0.2+0.8\big(1-Y_3^{13}\big)\big)+Y_{11}^{14}\big(0.2+0.8\big(1-Y_3^{14}\big)\big)+Y_{11}^{15}\big(0.2+0.8\big(1-Y_3^{15}\big)\big)\Big]$$

$$+10\times Z_5^2\times 4\times 4\times\Big[Y_{10}^1\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^1\big)\big)+Y_{10}^2\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^2\big)\big)+Y_{10}^3\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^3\big)\big)+$$

$$Y_{10}^4\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^4\big)\big)+Y_{10}^5\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^5\big)\big)+Y_{10}^6\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^6\big)\big)+Y_{10}^7\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^7\big)\big)+$$

$$Y_{10}^8\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^8\big)\big)+Y_{10}^9\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^9\big)\big)+Y_{10}^{10}\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^{10}\big)\big)+Y_{10}^{11}\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^{11}\big)\big)+$$

$$Y_{10}^{12}\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^{12}\big)\big)+Y_{10}^{13}\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^{13}\big)\big)+Y_{10}^{14}\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^{14}\big)\big)+Y_{10}^{15}\big(0.2+0.8\big(1-X_{5,2}^{2,1}Y_2^{15}\big)\big)\Big]$$

$$+10\times Z_5^2\times 4\times 4\times\Big[Y_{10}^1\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^1\big)\big)+Y_{10}^2\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^2\big)\big)+Y_{10}^3\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^3\big)\big)+$$

$$Y_{10}^4\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^4\big)\big)+Y_{10}^5\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^5\big)\big)+Y_{10}^6\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^6\big)\big)+Y_{10}^7\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^7\big)\big)+$$

$$Y_{10}^8\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^8\big)\big)+Y_{10}^9\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^9\big)\big)+Y_{10}^{10}\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^{10}\big)\big)+Y_{10}^{11}\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^{11}\big)\big)+$$

$$Y_{10}^{12}\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^{12}\big)\big)+Y_{10}^{13}\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^{13}\big)\big)+Y_{10}^{14}\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^{14}\big)\big)+Y_{10}^{15}\big(0.2+0.8\big(1-X_{5,15}^{2,1}Y_{15}^{15}\big)\big)\Big]$$

$$+10\times Z_5^2\times 3\times 3\times\Big[Y_{11}^1\big(0.2+0.8\big(1-Y_{10}^1\big)\big)+Y_{11}^2\big(0.2+0.8\big(1-Y_{10}^2\big)\big)+Y_{11}^3\big(0.2+0.8\big(1-Y_{10}^3\big)\big)+$$

$$Y_{11}^4\big(0.2+0.8\big(1-Y_{10}^4\big)\big)+Y_{11}^5\big(0.2+0.8\big(1-Y_{10}^5\big)\big)+Y_{11}^6\big(0.2+0.8\big(1-Y_{10}^6\big)\big)+Y_{11}^7\big(0.2+0.8\big(1-Y_{10}^7\big)\big)+$$

$$Y_{11}^8\big(0.2+0.8\big(1-Y_{10}^8\big)\big)+Y_{11}^9\big(0.2+0.8\big(1-Y_{10}^9\big)\big)+Y_{11}^{10}\big(0.2+0.8\big(1-Y_{10}^{10}\big)\big)+Y_{11}^{11}\big(0.2+0.8\big(1-Y_{10}^{11}\big)\big)+$$

$$Y_{11}^{12}\big(0.2+0.8\big(1-Y_{10}^{12}\big)\big)+Y_{11}^{13}\big(0.2+0.8\big(1-Y_{10}^{13}\big)\big)+Y_{11}^{14}\big(0.2+0.8\big(1-Y_{10}^{14}\big)\big)+Y_{11}^{15}\big(0.2+0.8\big(1-Y_{10}^{15}\big)\big)\Big]$$

$$+10\times Z_6^1\times 4\times 4\times X_{6,1}^{1,2}\Big[Y_1^1\big(0.2+0.8\big(1-Y_5^1\big)\big)+Y_1^2\big(0.2+0.8\big(1-Y_5^2\big)\big)+Y_1^3\big(0.2+0.8\big(1-Y_5^3\big)\big)+$$

$$Y_1^4\big(0.2+0.8\big(1-Y_5^4\big)\big)+Y_1^5\big(0.2+0.8\big(1-Y_5^5\big)\big)+Y_1^6\big(0.2+0.8\big(1-Y_5^6\big)\big)+Y_1^7\big(0.2+0.8\big(1-Y_5^7\big)\big)+$$

$$Y_1^8\big(0.2+0.8\big(1-Y_5^8\big)\big)+Y_1^9\big(0.2+0.8\big(1-Y_5^9\big)\big)+Y_1^{10}\big(0.2+0.8\big(1-Y_5^{10}\big)\big)+Y_1^{11}\big(0.2+0.8\big(1-Y_5^{11}\big)\big)+$$

$$Y_1^{12}\big(0.2+0.8\big(1-Y_5^{12}\big)\big)+Y_1^{13}\big(0.2+0.8\big(1-Y_5^{13}\big)\big)+Y_1^{14}\big(0.2+0.8\big(1-Y_5^{14}\big)\big)+Y_1^{15}\big(0.2+0.8\big(1-Y_5^{15}\big)\big)\Big]$$

$$+10\times Z_6^1\times4\times4\times X_{6,14}^{1,2}\left[Y_{14}^1\left(0.2+0.8\left(1-Y_5^1\right)\right)+Y_{14}^2\left(0.2+0.8\left(1-Y_5^2\right)\right)+Y_{14}^3\left(0.2+0.8\left(1-Y_5^3\right)\right)+\right.$$

$$Y_{14}^4\left(0.2+0.8\left(1-Y_5^4\right)\right)+Y_{14}^5\left(0.2+0.8\left(1-Y_5^5\right)\right)+Y_{14}^6\left(0.2+0.8\left(1-Y_5^6\right)\right)+Y_{14}^7\left(0.2+0.8\left(1-Y_5^7\right)\right)+$$

$$Y_{14}^8\left(0.2+0.8\left(1-Y_5^8\right)\right)+Y_{14}^9\left(0.2+0.8\left(1-Y_5^9\right)\right)+Y_{14}^{10}\left(0.2+0.8\left(1-Y_5^{10}\right)\right)+Y_{14}^{11}\left(0.2+0.8\left(1-Y_5^{11}\right)\right)+$$

$$\left.Y_{14}^{12}\left(0.2+0.8\left(1-Y_5^{12}\right)\right)+Y_{14}^{13}\left(0.2+0.8\left(1-Y_5^{13}\right)\right)+Y_{14}^{14}\left(0.2+0.8\left(1-Y_5^{14}\right)\right)+Y_{14}^{15}\left(0.2+0.8\left(1-Y_5^{15}\right)\right)\right]$$

$$+10\times Z_6^2\times4\times4$$

$$\left[Y_3^1\left(0.2+0.8\left(1-Y_4^1\right)\right)+Y_3^2\left(0.2+0.8\left(1-Y_4^2\right)\right)+Y_3^3\left(0.2+0.8\left(1-Y_4^3\right)\right)+\right.$$

$$Y_3^4\left(0.2+0.8\left(1-Y_4^4\right)\right)+Y_3^5\left(0.2+0.8\left(1-Y_4^5\right)\right)+Y_3^6\left(0.2+0.8\left(1-Y_4^6\right)\right)+Y_3^7\left(0.2+0.8\left(1-Y_4^7\right)\right)+$$

$$Y_3^8\left(0.2+0.8\left(1-Y_4^8\right)\right)+Y_3^9\left(0.2+0.8\left(1-Y_4^9\right)\right)+Y_3^{10}\left(0.2+0.8\left(1-Y_4^{10}\right)\right)+Y_3^{11}\left(0.2+0.8\left(1-Y_4^{11}\right)\right)+$$

$$\left.Y_3^{12}\left(0.2+0.8\left(1-Y_4^{12}\right)\right)+Y_3^{13}\left(0.2+0.8\left(1-Y_4^{13}\right)\right)+Y_3^{14}\left(0.2+0.8\left(1-Y_4^{14}\right)\right)+Y_3^{15}\left(0.2+0.8\left(1-Y_4^{15}\right)\right)\right]$$

$$+10\times Z_7^1\times4\times4\times\left[Y_4^1\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^1\right)\right)+Y_4^2\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^2\right)\right)+Y_4^3\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^3\right)\right)+\right.$$

$$Y_4^4\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^4\right)\right)+Y_4^5\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^5\right)\right)+Y_4^6\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^6\right)\right)+Y_4^7\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^7\right)\right)+$$

$$Y_4^8\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^8\right)\right)+Y_4^9\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^9\right)\right)+Y_4^{10}\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^{10}\right)\right)+Y_4^{11}\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^{11}\right)\right)+$$

$$\left.Y_4^{12}\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^{12}\right)\right)+Y_4^{13}\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^{13}\right)\right)+Y_4^{14}\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^{14}\right)\right)+Y_4^{15}\left(0.2+0.8\left(1-X_{7,2}^{1,1}Y_2^{15}\right)\right)\right]$$

$$+10\times Z_7^1\times4\times4\times\left[Y_4^1\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^1\right)\right)+Y_4^2\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^2\right)\right)+Y_4^3\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^3\right)\right)+\right.$$

$$Y_4^4\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^4\right)\right)+Y_4^5\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^5\right)\right)+Y_4^6\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^6\right)\right)+Y_4^7\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^7\right)\right)+$$

$$Y_4^8\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^8\right)\right)+Y_4^9\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^9\right)\right)+Y_4^{10}\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^{10}\right)\right)+Y_4^{11}\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^{11}\right)\right)+$$

$$\left.Y_4^{12}\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^{12}\right)\right)+Y_4^{13}\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^{13}\right)\right)+Y_4^{14}\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^{14}\right)\right)+Y_4^{15}\left(0.2+0.8\left(1-X_{7,15}^{1,1}Y_{15}^{15}\right)\right)\right]$$

$$+10\times Z_7^1\times3\times3\times Y_{11}^1\left(0.2+0.8\left(1-Y_4^1\right)\right)+Y_{11}^2\left(0.2+0.8\left(1-Y_4^2\right)\right)+Y_{11}^3\left(0.2+0.8\left(1-Y_4^3\right)\right)+$$

$$Y_{11}^4\left(0.2+0.8\left(1-Y_4^4\right)\right)+Y_{11}^5\left(0.2+0.8\left(1-Y_4^5\right)\right)+Y_{11}^6\left(0.2+0.8\left(1-Y_4^6\right)\right)+Y_{11}^7\left(0.2+0.8\left(1-Y_4^7\right)\right)+$$

$$Y_{11}^8\left(0.2+0.8\left(1-Y_4^8\right)\right)+Y_{11}^9\left(0.2+0.8\left(1-Y_4^9\right)\right)+Y_{11}^{10}\left(0.2+0.8\left(1-Y_4^{10}\right)\right)+Y_{11}^{11}\left(0.2+0.8\left(1-Y_4^{11}\right)\right)+$$

$$Y_{11}^{12}\left(0.2+0.8\left(1-Y_4^{12}\right)\right)+Y_{11}^{13}\left(0.2+0.8\left(1-Y_4^{13}\right)\right)+Y_{11}^{14}\left(0.2+0.8\left(1-Y_4^{14}\right)\right)+Y_{11}^{15}\left(0.2+0.8\left(1-Y_4^{15}\right)\right)$$

$$+10\times Z_7^2\times4\times4\times X_{7,1}^{2,2}\left[Y_1^1\left(0.2+0.8\left(1-Y_4^1\right)\right)+Y_1^2\left(0.2+0.8\left(1-Y_4^2\right)\right)+Y_1^3\left(0.2+0.8\left(1-Y_4^3\right)\right)+\right.$$

$$Y_1^4\left(0.2+0.8\left(1-Y_4^4\right)\right)+Y_1^5\left(0.2+0.8\left(1-Y_4^5\right)\right)+Y_1^6\left(0.2+0.8\left(1-Y_4^6\right)\right)+Y_1^7\left(0.2+0.8\left(1-Y_4^7\right)\right)+$$

$$Y_1^8\left(0.2+0.8\left(1-Y_4^8\right)\right)+Y_1^9\left(0.2+0.8\left(1-Y_4^9\right)\right)+Y_1^{10}\left(0.2+0.8\left(1-Y_4^{10}\right)\right)+Y_1^{11}\left(0.2+0.8\left(1-Y_4^{11}\right)\right)+$$

$$\left.Y_1^{12}\left(0.2+0.8\left(1-Y_4^{12}\right)\right)+Y_1^{13}\left(0.2+0.8\left(1-Y_4^{13}\right)\right)+Y_1^{14}\left(0.2+0.8\left(1-Y_4^{14}\right)\right)+Y_1^{15}\left(0.2+0.8\left(1-Y_4^{15}\right)\right)\right]$$

$$+10\times Z_7^2\times 4\times 4\times X^{2,2}_{7,14}\Big[Y^1_{14}\big(0.2+0.8\big(1-Y^1_4\big)\big)+Y^2_{14}\big(0.2+0.8\big(1-Y^2_4\big)\big)+Y^3_{14}\big(0.2+0.8\big(1-Y^3_4\big)\big)+$$

$$Y^4_{14}\big(0.2+0.8\big(1-Y^4_4\big)\big)+Y^5_{14}\big(0.2+0.8\big(1-Y^5_4\big)\big)+Y^6_{14}\big(0.2+0.8\big(1-Y^6_4\big)\big)+Y^7_{14}\big(0.2+0.8\big(1-Y^7_4\big)\big)+$$

$$Y^8_{14}\big(0.2+0.8\big(1-Y^8_4\big)\big)+Y^9_{14}\big(0.2+0.8\big(1-Y^9_4\big)\big)+Y^{10}_{14}\big(0.2+0.8\big(1-Y^{10}_4\big)\big)+Y^{11}_{14}\big(0.2+0.8\big(1-Y^{11}_4\big)\big)+$$

$$Y^{12}_{14}\big(0.2+0.8\big(1-Y^{12}_4\big)\big)+Y^{13}_{14}\big(0.2+0.8\big(1-Y^{13}_4\big)\big)+Y^{14}_{14}\big(0.2+0.8\big(1-Y^{14}_4\big)\big)+Y^{15}_{14}\big(0.2+0.8\big(1-Y^{15}_4\big)\big)\Big]$$

$$+10\times Z_7^2\times 3\times 3\times X^{2,3}_{7,2}$$

$$\Big[Y^1_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^1_1\big)\big)+Y^2_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^2_1\big)\big)+Y^3_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^3_1\big)\big)+$$

$$Y^4_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^4_1\big)\big)+Y^5_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^5_1\big)\big)+Y^6_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^6_1\big)\big)+Y^7_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^7_1\big)\big)+$$

$$Y^8_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^8_1\big)\big)+Y^9_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^9_1\big)\big)+Y^{10}_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{10}_1\big)\big)+Y^{11}_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{11}_1\big)\big)+$$

$$Y^{12}_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{12}_1\big)\big)+Y^{13}_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{13}_1\big)\big)+Y^{14}_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{14}_1\big)\big)+Y^{15}_2\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{15}_1\big)\big)\Big]$$

$$+10\times Z_7^2\times 3\times 3\times X^{2,3}_{7,15}$$

$$\Big[Y^1_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^1_1\big)\big)+Y^2_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^2_1\big)\big)+Y^3_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^3_1\big)\big)+$$

$$Y^4_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^4_1\big)\big)+Y^5_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^5_1\big)\big)+Y^6_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^6_1\big)\big)+Y^7_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^7_1\big)\big)+$$

$$Y^8_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^8_1\big)\big)+Y^9_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^9_1\big)\big)+Y^{10}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{10}_1\big)\big)+Y^{11}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{11}_1\big)\big)+$$

$$Y^{12}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{12}_1\big)\big)+Y^{13}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{13}_1\big)\big)+Y^{14}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{14}_1\big)\big)+Y^{15}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,1}Y^{15}_1\big)\big)\Big]$$

$$+10\times Z_7^2\times 3\times 3\times X^{2,3}_{7,2}$$

$$\Big[Y^1_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^1_{14}\big)\big)+Y^2_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^2_{14}\big)\big)+Y^3_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^3_{14}\big)\big)+$$

$$Y^4_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^4_{14}\big)\big)+Y^5_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^5_{14}\big)\big)+Y^6_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^6_{14}\big)\big)+Y^7_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^7_{14}\big)\big)+$$

$$Y^8_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^8_{14}\big)\big)+Y^9_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^9_{14}\big)\big)+Y^{10}_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{10}_{14}\big)\big)+Y^{11}_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{11}_{14}\big)\big)+$$

$$Y^{12}_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{12}_{14}\big)\big)+Y^{13}_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{13}_{14}\big)\big)+Y^{14}_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{14}_{14}\big)\big)+Y^{15}_2\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{15}_{14}\big)$$

$$+10\times Z_7^2\times 3\times 3\times X^{2,3}_{7,15}$$

$$\Big[Y^1_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^1_{14}\big)\big)+Y^2_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^2_{14}\big)\big)+Y^3_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^3_{14}\big)\big)+$$

$$Y^4_{15}\big(0.2+0.8\big(1-Y^4_{14}\big)\big)+Y^5_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^5_{14}\big)\big)+Y^6_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^6_{14}\big)\big)+Y^7_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^7_{14}\big)\big)+$$

$$Y^8_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^8_{14}\big)\big)+Y^9_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^9_{14}\big)\big)+Y^{10}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{10}_{14}\big)\big)+Y^{11}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{11}_{14}\big)\big)+$$

$$Y^{12}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{12}_{14}\big)\big)+Y^{13}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{13}_{14}\big)\big)+Y^{14}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{14}_{14}\big)\big)+Y^{15}_{15}\big(0.2+0.8\big(1-X^{2,2}_{7,14}Y^{15}_{14}$$

$$+10\times Z_8^1\times 4\times 4\times X^{1,2}_{8,1}\Big[Y^1_1\big(0.2+0.8\big(1-Y^1_{10}\big)\big)+Y^2_1\big(0.2+0.8\big(1-Y^2_{10}\big)\big)+Y^3_1\big(0.2+0.8\big(1-Y^3_{10}\big)\big)+$$

$$Y_1^4\left(0.2+0.8\left(1-Y_{10}^4\right)\right)+Y_1^5\left(0.2+0.8\left(1-Y_{10}^5\right)\right)+Y_1^6\left(0.2+0.8\left(1-Y_{10}^6\right)\right)+Y_1^7\left(0.2+0.8\left(1-Y_{10}^7\right)\right)+$$
$$Y_1^8\left(0.2+0.8\left(1-Y_{10}^8\right)\right)+Y_1^9\left(0.2+0.8\left(1-Y_{10}^9\right)\right)+Y_1^{10}\left(0.2+0.8\left(1-Y_{10}^{10}\right)\right)+Y_1^{11}\left(0.2+0.8\left(1-Y_{10}^{11}\right)\right)+$$
$$Y_1^{12}\left(0.2+0.8\left(1-Y_{10}^{12}\right)\right)+Y_1^{13}\left(0.2+0.8\left(1-Y_{10}^{13}\right)\right)+Y_1^{14}\left(0.2+0.8\left(1-Y_{10}^{14}\right)\right)+Y_1^{15}\left(0.2+0.8\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_8^1\times4\times4\times X_{8,1}^{1,2}\Big[Y_{14}^1\left(0.2+0.8\left(1-Y_{10}^1\right)\right)+Y_{14}^2\left(0.2+0.8\left(1-Y_{10}^2\right)\right)+Y_{14}^3\left(0.2+0.8\left(1-Y_{10}^3\right)\right)+$$
$$Y_{14}^4\left(0.2+0.8\left(1-Y_{10}^4\right)\right)+Y_{14}^5\left(0.2+0.8\left(1-Y_{10}^5\right)\right)+Y_{14}^6\left(0.2+0.8\left(1-Y_{10}^6\right)\right)+Y_{14}^7\left(0.2+0.8\left(1-Y_{10}^7\right)\right)+$$
$$Y_{14}^8\left(0.2+0.8\left(1-Y_{10}^8\right)\right)+Y_{14}^9\left(0.2+0.8\left(1-Y_{10}^9\right)\right)+Y_{14}^{10}\left(0.2+0.8\left(1-Y_{10}^{10}\right)\right)+Y_{14}^{11}\left(0.2+0.8\left(1-Y_{10}^{11}\right)\right)+$$
$$Y_{14}^{12}\left(0.2+0.8\left(1-Y_{10}^{12}\right)\right)+Y_{14}^{13}\left(0.2+0.8\left(1-Y_{10}^{13}\right)\right)+Y_{14}^{14}\left(0.2+0.8\left(1-Y_{10}^{14}\right)\right)+Y_{14}^{15}\left(0.2+0.8\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_8^2\times4\times4\times\Big[Y_{10}^1\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^1\right)\right)+Y_{10}^2\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^2\right)\right)+Y_{10}^3\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^3\right)\right)+$$
$$Y_{10}^4\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^4\right)\right)+Y_{10}^5\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^5\right)\right)+Y_{10}^6\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^6\right)\right)+Y_{10}^7\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^7\right)\right)+$$
$$Y_{10}^8\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^8\right)\right)+Y_{10}^9\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^9\right)\right)+Y_{10}^{10}\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^{10}\right)\right)+Y_{10}^{11}\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^{11}\right)\right)+$$
$$Y_{10}^{12}\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^{12}\right)\right)+Y_{10}^{13}\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^{13}\right)\right)+Y_{10}^{14}\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^{14}\right)\right)+Y_{10}^{15}\left(0.2+0.8\left(1-X_{8,2}^{2,1}Y_2^{15}\right)\right)\Big]$$

$$+10\times Z_8^2\times4\times4\times\Big[Y_{10}^1\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^1\right)\right)+Y_{10}^2\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^2\right)\right)+Y_{10}^3\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^3\right)\right)+$$
$$Y_{10}^4\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^4\right)\right)+Y_{10}^5\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^5\right)\right)+Y_{10}^6\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^6\right)\right)+Y_{10}^7\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^7\right)\right)+$$
$$Y_{10}^8\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^8\right)\right)+Y_{10}^9\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^9\right)\right)+Y_{10}^{10}\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^{10}\right)\right)+Y_{10}^{11}\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^{11}\right)\right)+$$
$$Y_{10}^{12}\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^{12}\right)\right)+Y_{10}^{13}\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^{13}\right)\right)+Y_{10}^{14}\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^{14}\right)\right)+Y_{10}^{15}\left(0.2+0.8\left(1-X_{8,15}^{2,1}Y_{15}^{15}\right)\right)\Big]$$

$$+10\times Z_8^2\times3\times3\times X_{8,1}^{2,3}\Big[Y_1^1\left(0.2+0.8\left(1-Y_{10}^1\right)\right)+Y_1^2\left(0.2+0.8\left(1-Y_{10}^2\right)\right)+Y_1^3\left(0.2+0.8\left(1-Y_{10}^3\right)\right)+$$
$$Y_1^4\left(0.2+0.8\left(1-Y_{10}^4\right)\right)+Y_1^5\left(0.2+0.8\left(1-Y_{10}^5\right)\right)+Y_1^6\left(0.2+0.8\left(1-Y_{10}^6\right)\right)+Y_1^7\left(0.2+0.8\left(1-Y_{10}^7\right)\right)+$$
$$Y_1^8\left(0.2+0.8\left(1-Y_{10}^8\right)\right)+Y_1^9\left(0.2+0.8\left(1-Y_{10}^9\right)\right)+Y_1^{10}\left(0.2+0.8\left(1-Y_{10}^{10}\right)\right)+Y_1^{11}\left(0.2+0.8\left(1-Y_{10}^{11}\right)\right)+$$
$$Y_1^{12}\left(0.2+0.8\left(1-Y_{10}^{12}\right)\right)+Y_1^{13}\left(0.2+0.8\left(1-Y_{10}^{13}\right)\right)+Y_1^{14}\left(0.2+0.8\left(1-Y_{10}^{14}\right)\right)+Y_1^{15}\left(0.2+0.8\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_8^2\times3\times3\times X_{8,14}^{2,3}\Big[Y_{14}^1\left(0.2+0.8\left(1-Y_{10}^1\right)\right)+Y_{14}^2\left(0.2+0.8\left(1-Y_{10}^2\right)\right)+Y_{14}^3\left(0.2+0.8\left(1-Y_{10}^3\right)\right)+$$
$$Y_{14}^4\left(0.2+0.8\left(1-Y_{10}^4\right)\right)+Y_{14}^5\left(0.2+0.8\left(1-Y_{10}^5\right)\right)+Y_{14}^6\left(0.2+0.8\left(1-Y_{10}^6\right)\right)+Y_{14}^7\left(0.2+0.8\left(1-Y_{10}^7\right)\right)+$$
$$Y_{14}^8\left(0.2+0.8\left(1-Y_{10}^8\right)\right)+Y_{14}^9\left(0.2+0.8\left(1-Y_{10}^9\right)\right)+Y_{14}^{10}\left(0.2+0.8\left(1-Y_{10}^{10}\right)\right)+Y_{14}^{11}\left(0.2+0.8\left(1-Y_{10}^{11}\right)\right)+$$
$$Y_{14}^{12}\left(0.2+0.8\left(1-Y_{10}^{12}\right)\right)+Y_{14}^{13}\left(0.2+0.8\left(1-Y_{10}^{13}\right)\right)+Y_{14}^{14}\left(0.2+0.8\left(1-Y_{10}^{14}\right)\right)+Y_{14}^{15}\left(0.2+0.8\left(1-Y_{10}^{15}\right)\right)\Big]$$

$$+10\times Z_9^1\times4\times4\Big[Y_7^1\left(0.2+0.8\left(1-Y_6^1\right)\right)+Y_7^2\left(0.2+0.8\left(1-Y_6^2\right)\right)+Y_7^3\left(0.2+0.8\left(1-Y_6^3\right)\right)+$$

$$Y_7^4\left(0.2+0.8\left(1-Y_6^4\right)\right)+Y_7^5\left(0.2+0.8\left(1-Y_6^5\right)\right)+Y_7^6\left(0.2+0.8\left(1-Y_6^6\right)\right)+Y_7^7\left(0.2+0.8\left(1-Y_6^7\right)\right)+$$

$$Y_7^8\left(0.2+0.8\left(1-Y_6^8\right)\right)+Y_7^9\left(0.2+0.8\left(1-Y_6^9\right)\right)+Y_7^{10}\left(0.2+0.8\left(1-Y_6^{10}\right)\right)+Y_7^{11}\left(0.2+0.8\left(1-Y_6^{11}\right)\right)+$$

$$Y_7^{12}\left(0.2+0.8\left(1-Y_6^{12}\right)\right)+Y_7^{13}\left(0.2+0.8\left(1-Y_6^{13}\right)\right)+Y_7^{14}\left(0.2+0.8\left(1-Y_6^{14}\right)\right)+Y_7^{15}\left(0.2+0.8\left(1-Y_6^{15}\right)\right)\Big]$$

$$+10\times Z_9^1\times 3\times 3\Big[Y_8^1\left(0.2+0.8\left(1-Y_7^1\right)\right)+Y_8^2\left(0.2+0.8\left(1-Y_7^2\right)\right)+Y_8^3\left(0.2+0.8\left(1-Y_7^3\right)\right)+$$

$$Y_8^4\left(0.2+0.8\left(1-Y_7^4\right)\right)+Y_8^5\left(0.2+0.8\left(1-Y_7^5\right)\right)+Y_8^6\left(0.2+0.8\left(1-Y_7^6\right)\right)+Y_8^7\left(0.2+0.8\left(1-Y_7^7\right)\right)+$$

$$Y_8^8\left(0.2+0.8\left(1-Y_7^8\right)\right)+Y_8^9\left(0.2+0.8\left(1-Y_7^9\right)\right)+Y_8^{10}\left(0.2+0.8\left(1-Y_7^{10}\right)\right)+Y_8^{11}\left(0.2-0.8\left(1-Y_7^{11}\right)\right)+$$

$$Y_8^{12}\left(0.2+0.8\left(1-Y_7^{12}\right)\right)+Y_8^{13}\left(0.2+0.8\left(1-Y_7^{13}\right)\right)+Y_8^{14}\left(0.2+0.8\left(1-Y_7^{14}\right)\right)+Y_8^{15}\left(0.2+0.8\left(1-Y_7^{15}\right)\right)\Big]$$

$$+10\times Z_9^2\times 4\times 4\times\Big[Y_6^1\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^1\right)\right)+Y_6^2\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^2\right)\right)+Y_6^3\left(0.2-0.8\left(1-X_{9,1}^{2,1}Y_1^3\right)\right)+$$

$$Y_6^4\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^4\right)\right)+Y_6^5\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^5\right)\right)+Y_6^6\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^6\right)\right)+Y_6^7\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^7\right)\right)+$$

$$Y_6^8\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^8\right)\right)+Y_6^9\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^9\right)\right)+Y_6^{10}\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^{10}\right)\right)+Y_6^{11}\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^{11}\right)\right)+$$

$$Y_6^{12}\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^{12}\right)\right)+Y_6^{13}\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^{13}\right)\right)+Y_6^{14}\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^{14}\right)\right)+Y_6^{15}\left(0.2+0.8\left(1-X_{9,1}^{2,1}Y_1^{15}\right)\right)\Big]$$

$$+10\times Z_9^2\times 4\times 4\times\Big[Y_6^1\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^1\right)\right)+Y_6^2\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^2\right)\right)+Y_6^3\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^3\right)\right)+$$

$$Y_6^4\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^4\right)\right)+Y_6^5\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^5\right)\right)+Y_6^6\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^6\right)\right)+Y_6^7\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^7\right)\right)+$$

$$Y_6^8\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^8\right)\right)+Y_6^9\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^9\right)\right)+Y_6^{10}\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^{10}\right)\right)+Y_6^{11}\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^{11}\right)\right)+$$

$$Y_6^{12}\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^{12}\right)\right)+Y_6^{13}\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^{13}\right)\right)+Y_6^{14}\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^{14}\right)\right)+Y_6^{15}\left(0.2+0.8\left(1-X_{9,14}^{2,1}Y_{14}^{15}\right)\right)$$

$$+10\times Z_9^2\times 3\times 3\times\Big[Y_8^1\left(0.2+0.8\left(1-Y_6^1\right)\right)+Y_8^2\left(0.2+0.8\left(1-Y_6^2\right)\right)+Y_8^3\left(0.2+0.8\left(1-Y_6^3\right)\right)+$$

$$Y_8^4\left(0.2+0.8\left(1-Y_6^4\right)\right)+Y_8^5\left(0.2+0.8\left(1-Y_6^5\right)\right)+Y_8^6\left(0.2+0.8\left(1-Y_6^6\right)\right)+Y_8^7\left(0.2-0.8\left(1-Y_6^7\right)\right)+$$

$$Y_8^8\left(0.2+0.8\left(1-Y_6^8\right)\right)+Y_8^9\left(0.2+0.8\left(1-Y_6^9\right)\right)+Y_8^{10}\left(0.2+0.8\left(1-Y_6^{10}\right)\right)+Y_8^{11}\left(0.2-0.8\left(1-Y_6^{11}\right)\right)+$$

$$Y_8^{12}\left(0.2+0.8\left(1-Y_6^{12}\right)\right)+Y_8^{13}\left(0.2+0.8\left(1-Y_6^{13}\right)\right)+Y_8^{14}\left(0.2+0.8\left(1-Y_6^{14}\right)\right)+Y_8^{15}\left(0.2-0.8\left(1-Y_6^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^1\times 4\times 4\times\Big[Y_4^1\left(0.2+0.8\left(1-Y_9^1\right)\right)+Y_4^2\left(0.2+0.8\left(1-Y_9^2\right)\right)+Y_4^3\left(0.2+0.8\left(1-Y_9^3\right)\right)+$$

$$Y_4^4\left(0.2+0.8\left(1-Y_9^4\right)\right)+Y_4^5\left(0.2+0.8\left(1-Y_9^5\right)\right)+Y_4^6\left(0.2+0.8\left(1-Y_9^6\right)\right)+Y_4^7\left(0.2+0.8\left(1-Y_9^7\right)\right)+$$

$$Y_4^8\left(0.2+0.8\left(1-Y_9^8\right)\right)+Y_4^9\left(0.2+0.8\left(1-Y_9^9\right)\right)+Y_4^{10}\left(0.2+0.8\left(1-Y_9^{10}\right)\right)+Y_4^{11}\left(0.2-0.8\left(1-Y_9^{11}\right)\right)+$$

$$Y_4^{12}\left(0.2+0.8\left(1-Y_9^{12}\right)\right)+Y_4^{13}\left(0.2+0.8\left(1-Y_9^{13}\right)\right)+Y_4^{14}\left(0.2+0.8\left(1-Y_9^{14}\right)\right)+Y_4^{15}\left(0.2-0.8\left(1-Y_9^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^1\times 3\times 3\times X_{10,1}^{1,2}\Big[Y_1^1\left(0.2+0.8\left(1-Y_4^1\right)\right)+Y_1^2\left(0.2+0.8\left(1-Y_4^2\right)\right)+Y_1^3\left(0.2-0.8\left(1-Y_4^3\right)\right)+$$

$$Y_1^4\left(0.2+0.8\left(1-Y_4^4\right)\right)+Y_1^5\left(0.2+0.8\left(1-Y_4^5\right)\right)+Y_1^6\left(0.2+0.8\left(1-Y_4^6\right)\right)+Y_1^7\left(0.2+0.8\left(1-Y_4^7\right)\right)+$$
$$Y_1^8\left(0.2+0.8\left(1-Y_4^8\right)\right)+Y_1^9\left(0.2+0.8\left(1-Y_4^9\right)\right)+Y_1^{10}\left(0.2+0.8\left(1-Y_4^{10}\right)\right)+Y_1^{11}\left(0.2+0.8\left(1-Y_4^{11}\right)\right)+$$
$$Y_1^{12}\left(0.2+0.8\left(1-Y_4^{12}\right)\right)+Y_1^{13}\left(0.2+0.8\left(1-Y_4^{13}\right)\right)+Y_1^{14}\left(0.2+0.8\left(1-Y_4^{14}\right)\right)+Y_1^{15}\left(0.2+0.8\left(1-Y_4^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^1\times 3\times 3\times X_{10,14}^{1,2}\Big[Y_{14}^1\left(0.2+0.8\left(1-Y_4^1\right)\right)+Y_{14}^2\left(0.2+0.8\left(1-Y_4^2\right)\right)+Y_{14}^3\left(0.2+0.8\left(1-Y_4^3\right)\right)+$$

$$Y_{14}^4\left(0.2+0.8\left(1-Y_4^4\right)\right)+Y_{14}^5\left(0.2+0.8\left(1-Y_4^5\right)\right)+Y_{14}^6\left(0.2+0.8\left(1-Y_4^6\right)\right)+Y_{14}^7\left(0.2+0.8\left(1-Y_4^7\right)\right)+$$
$$Y_{14}^8\left(0.2+0.8\left(1-Y_4^8\right)\right)+Y_{14}^9\left(0.2+0.8\left(1-Y_4^9\right)\right)+Y_{14}^{10}\left(0.2+0.8\left(1-Y_4^{10}\right)\right)+Y_{14}^{11}\left(0.2+0.8\left(1-Y_4^{11}\right)\right)+$$
$$Y_{14}^{12}\left(0.2+0.8\left(1-Y_4^{12}\right)\right)+Y_{14}^{13}\left(0.2+0.8\left(1-Y_4^{13}\right)\right)+Y_{14}^{14}\left(0.2+0.8\left(1-Y_4^{14}\right)\right)+Y_{14}^{15}\left(0.2+0.8\left(1-Y_4^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^2\times 4\times 4\Big[Y_6^1\left(0.2+0.8\left(1-Y_8^1\right)\right)+Y_6^2\left(0.2+0.8\left(1-Y_8^2\right)\right)+Y_6^3\left(0.2+0.8\left(1-Y_8^3\right)\right)+$$

$$Y_6^4\left(0.2+0.8\left(1-Y_8^4\right)\right)+Y_6^5\left(0.2+0.8\left(1-Y_8^{51}\right)\right)+Y_6^6\left(0.2+0.8\left(1-Y_8^6\right)\right)+Y_6^7\left(0.2+0.8\left(1-Y_8^7\right)\right)+$$
$$Y_6^8\left(0.2+0.8\left(1-Y_8^{81}\right)\right)+Y_6^9\left(0.2+0.8\left(1-Y_8^{91}\right)\right)+Y_6^{10}\left(0.2+0.8\left(1-Y_8^{10}\right)\right)+Y_6^{11}\left(0.2+0.8\left(1-Y_8^{11}\right)\right)+$$

$$Y_6^{12}\left(0.2+0.8\left(1-Y_8^{12}\right)\right)+Y_6^{13}\left(0.2+0.8\left(1-Y_8^{13}\right)\right)+Y_6^{14}\left(0.2+0.8\left(1-Y_8^{14}\right)\right)+Y_6^{15}\left(0.2+0.8\left(1-Y_8^{15}\right)\right)\Big]$$

$$+10\times Z_{10}^2\times 3\times 3$$

$$\Big[Y_5^1\left(0.2+0.8\left(1-Y_6^1\right)\right)+Y_5^2\left(0.2+0.8\left(1-Y_6^2\right)\right)+Y_5^3\left(0.2+0.8\left(1-Y_6^3\right)\right)+$$
$$Y_5^4\left(0.2+0.8\left(1-Y_6^4\right)\right)+Y_5^5\left(0.2+0.8\left(1-Y_6^5\right)\right)+Y_5^6\left(0.2+0.8\left(1-Y_6^6\right)\right)+Y_5^7\left(0.2+0.8\left(1-Y_6^7\right)\right)+$$
$$Y_5^8\left(0.2+0.8\left(1-Y_6^8\right)\right)+Y_5^9\left(0.2+0.8\left(1-Y_6^9\right)\right)+Y_5^{10}\left(0.2+0.8\left(1-Y_6^{10}\right)\right)+Y_5^{11}\left(0.2+0.8\left(1-Y_6^{11}\right)\right)+$$
$$Y_5^{12}\left(0.2+0.8\left(1-Y_6^{12}\right)\right)+Y_5^{13}\left(0.2+0.8\left(1-Y_6^{13}\right)\right)+Y_5^{14}\left(0.2+0.8\left(1-Y_6^{14}\right)\right)+Y_5^{15}\left(0.2+0.8\left(1-Y_6^{15}\right)\right)\Big]$$

A-2.2 *Constraints :*

(a) Assignment of process plan to a part

$Z_1^1+Z_1^2=1$; (For part 1)

$Z_2^1+Z_2^2=1$; (For part 2)

$Z_3^1+Z_3^2=1$; (For part 3)

$Z_4^1+Z_4^2=1$; (For part 4)

$Z_5^1+Z_5^2=1$; (For part 5)

$Z_6^1+Z_6^2=1$; (For part 6)

$Z_7^1+Z_7^2=1$; (For part 7)

$Z_8^1+Z_8^2=1$; (For part 8)

$Z_9^1+Z_9^2=1$; (For part 9)

$Z_{10}^1+Z_{10}^2=10$; (For part 10)

(b) Assignment of machines to a cell

$Y_1^1 + Y_1^2 + Y_1^3 + Y_1^4 + Y_1^5 + Y_1^6 + Y_1^7 + Y_1^8 + Y_1^9 + Y_1^{10} + Y_1^{11} + Y_1^{12} + Y_1^{13} + Y_1^{14} + Y_1^{15} = 1$; (For machine 1)

$Y_2^1 + Y_2^2 + Y_2^3 + Y_2^4 + Y_2^5 + Y_2^6 + Y_2^7 + Y_2^8 + Y_2^9 + Y_2^{10} + Y_2^{11} + Y_2^{12} + Y_2^{13} + Y_2^{14} + Y_2^{15} = 1$; (For machine 2)

$Y_3^1 + Y_3^2 + Y_3^3 + Y_3^4 + Y_3^5 + Y_3^6 + Y_3^7 + Y_3^8 + Y_3^9 + Y_3^{10} + Y_3^{11} + Y_3^{12} + Y_3^{13} + Y_3^{14} + Y_3^{15} = 1$; (For machine 3)

$Y_4^1 + Y_4^2 + Y_4^3 + Y_4^4 + Y_4^5 + Y_4^6 + Y_4^7 + Y_4^8 + Y_4^9 + Y_4^{10} + Y_4^{11} + Y_4^{12} + Y_4^{13} + Y_4^{14} + Y_4^{15} = 1$; (For machine 4)

$Y_5^1 + Y_5^2 + Y_5^3 + Y_5^4 + Y_5^5 + Y_5^6 + Y_5^7 + Y_5^8 + Y_5^9 + Y_5^{10} + Y_5^{11} + Y_5^{12} + Y_5^{13} + Y_5^{14} + Y_5^{15} = 1$; (For machine 5)

$Y_6^1 + Y_6^2 + Y_6^3 + Y_6^4 + Y_6^5 + Y_6^6 + Y_6^7 + Y_6^8 + Y_6^9 + Y_6^{10} + Y_6^{11} + Y_6^{12} + Y_6^{13} + Y_6^{14} + Y_6^{15} = 1$; (For machine 6)

$Y_7^1 + Y_7^2 + Y_7^3 + Y_7^4 + Y_7^5 + Y_7^6 + Y_7^7 + Y_7^8 + Y_7^9 + Y_7^{10} + Y_7^{11} + Y_7^{12} + Y_7^{13} + Y_7^{14} + Y_7^{15} = 1$; (For machine 7)

$Y_8^1 + Y_8^2 + Y_8^3 + Y_8^4 + Y_8^5 + Y_8^6 + Y_8^7 + Y_8^8 + Y_8^9 + Y_8^{10} + Y_8^{11} + Y_8^{12} + Y_8^{13} + Y_8^{14} + Y_8^{15} = 1$; (For machine 8)

$Y_9^1 + Y_9^2 + Y_9^3 + Y_9^4 + Y_9^5 + Y_9^6 + Y_9^7 + Y_9^8 + Y_9^9 + Y_9^{10} + Y_9^{11} + Y_9^{12} + Y_9^{13} + Y_9^{14} + Y_9^{15} = 1$; (For machine 9)

$Y_{10}^1 + Y_{10}^2 + Y_{10}^3 + Y_{10}^4 + Y_{10}^5 + Y_{10}^6 + Y_{10}^7 + Y_{10}^8 + Y_{10}^9 + Y_{10}^{10} + Y_{10}^{11} + Y_{10}^{12} + Y_{10}^{13} + Y_{10}^{14} + Y_{10}^{15} = 1$; (For machine 10)

$Y_{11}^1 + Y_{11}^2 + Y_{11}^3 + Y_{11}^4 + Y_{11}^5 + Y_{11}^6 + Y_{11}^7 + Y_{11}^8 + Y_{11}^9 + Y_{11}^{10} + Y_{11}^{11} + Y_{11}^{12} + Y_{11}^{13} + Y_{11}^{14} + Y_{11}^{15} = 1$; (For machine 11)

$Y_{12}^1 + Y_{12}^2 + Y_{12}^3 + Y_{12}^4 + Y_{12}^5 + Y_{12}^6 + Y_{12}^7 + Y_{12}^8 + Y_{12}^9 + Y_{12}^{10} + Y_{12}^{11} + Y_{12}^{12} + Y_{12}^{13} + Y_{12}^{14} + Y_{12}^{15} = 1$; (For machine 12)

$Y_{13}^1 + Y_{13}^2 + Y_{13}^3 + Y_{13}^4 + Y_{13}^5 + Y_{13}^6 + Y_{13}^7 + Y_{13}^8 + Y_{13}^9 + Y_{13}^{10} + Y_{13}^{11} + Y_{13}^{12} + Y_{13}^{13} + Y_{13}^{14} + Y_{13}^{15} = 1$; (For machine 13)

$Y_{14}^1 + Y_{14}^2 + Y_{14}^3 + Y_{14}^4 + Y_{14}^5 + Y_{14}^6 + Y_{14}^7 + Y_{14}^8 + Y_{14}^9 + Y_{14}^{10} + Y_{14}^{11} + Y_{14}^{12} + Y_{14}^{13} + Y_{14}^{14} + Y_{14}^{15} = 1$; (For machine 14)

$Y_{15}^1 + Y_{15}^2 + Y_{15}^3 + Y_{15}^4 + Y_{15}^5 + Y_{15}^6 + Y_{15}^7 + Y_{15}^8 + Y_{15}^9 + Y_{15}^{10} + Y_{15}^{11} + Y_{15}^{12} + Y_{15}^{13} + Y_{15}^{14} + Y_{15}^{15} = 1$; (For machine 15)

(c) Limitation on cell size

$Y_1^1 + Y_2^1 + Y_3^1 + Y_4^1 + Y_5^1 + Y_6^1 + Y_7^1 + Y_8^1 + Y_9^1 + Y_{10}^1 + Y_{11}^1 + Y_{12}^1 + Y_{13}^1 + Y_{14}^1 + Y_{15}^1 \leq 5$; (For cell 1)

$Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 + Y_6^2 + Y_7^2 + Y_8^2 + Y_9^2 + Y_{10}^2 + Y_{11}^2 + Y_{12}^2 + Y_{13}^2 + Y_{14}^2 + Y_{15}^2 \leq 5$; (For cell 2)

$Y_1^3 + Y_2^3 + Y_3^3 + Y_4^3 + Y_5^3 + Y_6^3 + Y_7^3 + Y_8^3 + Y_9^3 + Y_{10}^3 + Y_{11}^3 + Y_{12}^3 + Y_{13}^3 + Y_{14}^3 + Y_{15}^3 \leq 5$; (For cell 3)

$Y_1^4 + Y_2^4 + Y_3^4 + Y_4^4 + Y_5^4 + Y_6^4 + Y_7^4 + Y_8^4 + Y_9^4 + Y_{10}^4 + Y_{11}^4 + Y_{12}^4 + Y_{13}^4 + Y_{14}^4 + Y_{15}^4 \leq 5$; (For cell 4)

$Y_1^5 + Y_2^5 + Y_3^5 + Y_4^5 + Y_5^5 + Y_6^5 + Y_7^5 + Y_8^5 + Y_9^5 + Y_{10}^5 + Y_{11}^5 + Y_{12}^5 + Y_{13}^5 + Y_{14}^5 + Y_{15}^5 \leq 5$; (For cell 5)

$Y_1^6 + Y_2^6 + Y_3^6 + Y_4^6 + Y_5^6 + Y_6^6 + Y_7^6 + Y_8^6 + Y_9^6 + Y_{10}^6 + Y_{11}^6 + Y_{12}^6 + Y_{13}^6 + Y_{14}^6 + Y_{15}^6 \leq 5$; (For cell 6)

$Y_1^7 + Y_2^7 + Y_3^7 + Y_4^7 + Y_5^7 + Y_6^7 + Y_7^7 + Y_8^7 + Y_9^7 + Y_{10}^7 + Y_{11}^7 + Y_{12}^7 + Y_{13}^7 + Y_{14}^7 + Y_{15}^7 \leq 5$; (For cell 7)

$Y_1^8 + Y_2^8 + Y_3^8 + Y_4^8 + Y_5^8 + Y_6^8 + Y_7^8 + Y_8^8 + Y_9^8 + Y_{10}^8 + Y_{11}^8 + Y_{12}^8 + Y_{13}^8 + Y_{14}^8 + Y_{15}^8 \leq 5$; (For cell 8)

$Y_1^9 + Y_2^9 + Y_3^9 + Y_4^9 + Y_5^9 + Y_6^9 + Y_7^9 + Y_8^9 + Y_9^9 + Y_{10}^9 + Y_{11}^9 + Y_{12}^9 + Y_{13}^9 + Y_{14}^9 + Y_{15}^9 \leq 5$; (For cell 9)

$Y_1^{10} + Y_2^{10} + Y_3^{10} + Y_4^{10} + Y_5^{10} + Y_6^{10} + Y_7^{10} + Y_8^{10} + Y_9^{10} + Y_{10}^{10} + Y_{11}^{10} + Y_{12}^{10} + Y_{13}^{10} + Y_{14}^{10} + Y_{15}^{10} \leq 5$; (For cell 10)

$Y_1^{11} + Y_2^{11} + Y_3^{11} + Y_4^{11} + Y_5^{11} + Y_6^{11} + Y_7^{11} + Y_8^{11} + Y_9^{11} + Y_{10}^{11} + Y_{11}^{11} + Y_{12}^{11} + Y_{13}^{11} + Y_{14}^{11} + Y_{15}^{11} \leq 5$; (For cell 11)

$Y_1^{12} + Y_2^{12} + Y_3^{12} + Y_4^{12} + Y_5^{12} + Y_6^{12} + Y_7^{12} + Y_8^{12} + Y_9^{12} + Y_{10}^{12} + Y_{11}^{12} + Y_{12}^{12} + Y_{13}^{12} + Y_{14}^{12} + Y_{15}^{12} \leq 5$; (For cell 12)

$Y_1^{13} + Y_2^{13} + Y_3^{13} + Y_4^{13} + Y_5^{13} + Y_6^{13} + Y_7^{13} + Y_8^{13} + Y_9^{13} + Y_{10}^{13} + Y_{11}^{13} + Y_{12}^{13} + Y_{13}^{13} + Y_{14}^{13} + Y_{15}^{13} \leq 5$; (For cell 13)

$Y_1^{14} + Y_2^{14} + Y_3^{14} + Y_4^{14} + Y_5^{14} + Y_6^{14} + Y_7^{14} + Y_8^{14} + Y_9^{14} + Y_{10}^{14} + Y_{11}^{14} + Y_{12}^{14} + Y_{13}^{14} + Y_{14}^{14} + Y_{15}^{14} \leq 5$; (For cell 14)

$Y_1^{15} + Y_2^{15} + Y_3^{15} + Y_4^{15} + Y_5^{15} + Y_6^{15} + Y_7^{15} + Y_8^{15} + Y_9^{15} + Y_{10}^{15} + Y_{11}^{15} + Y_{12}^{15} + Y_{13}^{15} + Y_{14}^{15} + Y_{15}^{15} \leq 5$; (For cell 15)

(d) Assignment of machine to a part for specific operation

$$\sum_{m=1}^{15} X_{p,m}^{r,o} = 1 \qquad p = 1, \ldots, 10; \qquad r = 1, 2;$$

$$o = 1, \ldots, O_p^r.$$

(e) Cell machine type constraint

$Y_1^1 + Y_{14}^1 \le 1$ (For cell 1 and machine type 1)
$Y_2^1 + Y_{15}^1 \le 1$ (For cell 1 and machine type 2)
$Y_3^1 \le 1$ (For cell 1 and machine type 3)
$Y_4^1 \le 1$ (For cell 1 and machine type 4)
$Y_5^1 \le 1$ (For cell 1 and machine type 5)
$Y_6^1 \le 1$ (For cell 1 and machine type 6)
$Y_7^1 \le 1$ (For cell 1 and machine type 7)
$Y_8^1 \le 1$ (For cell 1 and machine type 8)
$Y_9^1 \le 1$ (For cell 1 and machine type 9)
$Y_{10}^1 \le 1$ (For cell 1 and machine type 10)
$Y_{11}^1 \le 1$ (For cell 1 and machine type 11)
$Y_{12}^1 \le 1$ (For cell 1 and machine type 12)
$Y_{13}^1 \le 1$ (For cell 1 and machine type 13)

$Y_1^2 + Y_{14}^2 \le 1$ (For cell 2 and machine type 1)
$Y_2^2 + Y_{15}^2 \le 1$ (For cell 2 and machine type 2)
$Y_3^2 \le 1$ (For cell 2 and machine type 3)
$Y_4^2 \le 1$ (For cell 2 and machine type 4)
$Y_5^2 \le 1$ (For cell 2 and machine type 5)
$Y_6^2 \le 1$ (For cell 2 and machine type 6)
$Y_7^2 \le 1$ (For cell 2 and machine type 7)
$Y_8^2 \le 1$ (For cell 2 and machine type 8)
$Y_9^2 \le 1$ (For cell 2 and machine type 9)
$Y_{10}^2 \le 1$ (For cell 2 and machine type 10)
$Y_{11}^2 \le 1$ (For cell 2 and machine type 11)
$Y_{12}^2 \le 1$ (For cell 2 and machine type 12)
$Y_{13}^2 \le 1$ (For cell 2 and machine type 13)

$Y_1^3 + Y_{14}^3 \le 1$ (For cell 3 and machine type 1)
$Y_2^3 + Y_{15}^3 \le 1$ (For cell 3 and machine type 2)
$Y_3^3 \le 1$ (For cell 3 and machine type 3)
$Y_4^3 \le 1$ (For cell 3 and machine type 4)
$Y_5^3 \le 1$ (For cell 3 and machine type 5)
$Y_6^3 \le 1$ (For cell 3 and machine type 6)
$Y_7^3 \le 1$ (For cell 3 and machine type 7)
$Y_8^3 \le 1$ (For cell 3 and machine type 8)
$Y_9^3 \le 1$ (For cell 3 and machine type 9)
$Y_{10}^3 \le 1$ (For cell 3 and machine type 10)
$Y_{11}^3 \le 1$ (For cell 3 and machine type 11)
$Y_{12}^3 \le 1$ (For cell 3 and machine type 12)
$Y_{13}^3 \le 1$ (For cell 3 and machine type 13)

$Y_1^4 + Y_{14}^4 \le 1$ (For cell 4 and machine type 1)
$Y_2^4 + Y_{15}^4 \le 1$ (For cell 4 and machine type 2)
$Y_3^4 \le 1$ (For cell 4 and machine type 3)
$Y_4^4 \le 1$ (For cell 4 and machine type 4)
$Y_5^4 \le 1$ (For cell 4 and machine type 5)
$Y_6^4 \le 1$ (For cell 4 and machine type 6)
$Y_7^4 \le 1$ (For cell 4 and machine type 7)
$Y_8^4 \le 1$ (For cell 4 and machine type 8)
$Y_9^4 \le 1$ (For cell 4 and machine type 9)
$Y_{10}^4 \le 1$ (For cell 4 and machine type 10)
$Y_{11}^4 \le 1$ (For cell 4 and machine type 11)
$Y_{12}^4 \le 1$ (For cell 4 and machine type 12)
$Y_{13}^4 \le 1$ (For cell 4 and machine type 13)

$Y_1^5 + Y_{14}^5 \leq 1$ (For cell 5 and machine type 1)
$Y_2^5 + Y_{15}^5 \leq 1$ (For cell 5 and machine type 2)
$Y_3^5 \leq 1$ (For cell 5 and machine type 3)
$Y_4^5 \leq 1$ (For cell 5 and machine type 4)
$Y_5^5 \leq 1$ (For cell 5 and machine type 5)
$Y_6^5 \leq 1$ (For cell 5 and machine type 6)
$Y_7^5 \leq 1$ (For cell 5 and machine type 7)
$Y_8^5 \leq 1$ (For cell 5 and machine type 8)
$Y_9^5 \leq 1$ (For cell 5 and machine type 9)
$Y_{10}^5 \leq 1$ (For cell 5 and machine type 10)
$Y_{11}^5 \leq 1$ (For cell 5 and machine type 11)
$Y_{12}^5 \leq 1$ (For cell 5 and machine type 12)
$Y_{13}^5 \leq 1$ (For cell 5 and machine type 13)

$Y_1^6 + Y_{14}^6 \leq 1$ (For cell 6 and machine type 1)
$Y_2^6 + Y_{15}^6 \leq 1$ (For cell 6 and machine type 2)
$Y_3^6 \leq 1$ (For cell 6 and machine type 3)
$Y_4^6 \leq 1$ (For cell 6 and machine type 4)
$Y_5^6 \leq 1$ (For cell 6 and machine type 5)
$Y_6^6 \leq 1$ (For cell 6 and machine type 6)
$Y_7^6 \leq 1$ (For cell 6 and machine type 7)
$Y_8^6 \leq 1$ (For cell 6 and machine type 8)
$Y_9^6 \leq 1$ (For cell 6 and machine type 9)
$Y_{10}^6 \leq 1$ (For cell 6 and machine type 10)
$Y_{11}^6 \leq 1$ (For cell 6 and machine type 11)
$Y_{12}^6 \leq 1$ (For cell 6 and machine type 12)
$Y_{13}^6 \leq 1$ (For cell 6 and machine type 13)

$Y_1^7 + Y_{14}^7 \leq 1$ (For cell 7 and machine type 1)
$Y_2^7 + Y_{15}^7 \leq 1$ (For cell 7 and machine type 2)
$Y_3^7 \leq 1$ (For cell 7 and machine type 3)
$Y_4^7 \leq 1$ (For cell 7 and machine type 4)
$Y_5^7 \leq 1$ (For cell 7 and machine type 5)
$Y_6^7 \leq 1$ (For cell 7 and machine type 6)
$Y_7^7 \leq 1$ (For cell 7 and machine type 7)
$Y_8^7 \leq 1$ (For cell 7 and machine type 8)
$Y_9^7 \leq 1$ (For cell 7 and machine type 9)
$Y_{10}^7 \leq 1$ (For cell 7 and machine type 10)
$Y_{11}^7 \leq 1$ (For cell 7 and machine type 11)
$Y_{12}^7 \leq 1$ (For cell 7 and machine type 12)
$Y_{13}^7 \leq 1$ (For cell 7 and machine type 13)

$Y_1^8 + Y_{14}^8 \leq 1$ (For cell 8 and machine type 1)
$Y_2^8 + Y_{15}^8 \leq 1$ (For cell 8 and machine type 2)
$Y_3^8 \leq 1$ (For cell 8 and machine type 3)
$Y_4^8 \leq 1$ (For cell 8 and machine type 4)
$Y_5^8 \leq 1$ (For cell 8 and machine type 5)
$Y_6^8 \leq 1$ (For cell 8 and machine type 6)
$Y_7^8 \leq 1$ (For cell 8 and machine type 7)
$Y_8^8 \leq 1$ (For cell 8 and machine type 8)
$Y_9^8 \leq 1$ (For cell 8 and machine type 9)
$Y_{10}^8 \leq 1$ (For cell 8 and machine type 10)
$Y_{11}^8 \leq 1$ (For cell 8 and machine type 11)
$Y_{12}^8 \leq 1$ (For cell 8 and machine type 12)
$Y_{13}^8 \leq 1$ (For cell 8 and machine type 13)

$Y_1^9 + Y_{14}^9 \leq 1$ (For cell 9 and machine type 1)
$Y_2^9 + Y_{15}^9 \leq 1$ (For cell 9 and machine type 2)
$Y_3^9 \leq 1$ (For cell 9 and machine type 3)
$Y_4^9 \leq 1$ (For cell 9 and machine type 4)
$Y_5^9 \leq 1$ (For cell 9 and machine type 5)
$Y_6^9 \leq 1$ (For cell 9 and machine type 6)
$Y_7^9 \leq 1$ (For cell 9 and machine type 7)
$Y_8^9 \leq 1$ (For cell 9 and machine type 8)
$Y_9^9 \leq 1$ (For cell 9 and machine type 9)
$Y_{10}^9 \leq 1$ (For cell 9 and machine type 10)
$Y_{11}^9 \leq 1$ (For cell 9 and machine type 11)
$Y_{12}^9 \leq 1$ (For cell 9 and machine type 12)
$Y_{13}^9 \leq 1$ (For cell 9 and machine type 13)

$Y_1^{10} + Y_{14}^{10} \leq 1$ (For cell 10 and machine type 1)
$Y_2^{10} + Y_{15}^{10} \leq 1$ (For cell 10 and machine type 2)
$Y_3^{10} \leq 1$ (For cell 10 and machine type 3)
$Y_4^{10} \leq 1$ (For cell 10 and machine type 4)
$Y_5^{10} \leq 1$ (For cell 10 and machine type 5)
$Y_6^{10} \leq 1$ (For cell 10 and machine type 6)
$Y_7^{10} \leq 1$ (For cell 10 and machine type 7)
$Y_8^{10} \leq 1$ (For cell 10 and machine type 8)
$Y_9^{10} \leq 1$ (For cell 10 and machine type 9)
$Y_{10}^{10} \leq 1$ (For cell 10 and machine type 10)
$Y_{11}^{10} \leq 1$ (For cell 10 and machine type 11)
$Y_{12}^{10} \leq 1$ (For cell 10 and machine type 12)
$Y_{13}^{10} \leq 1$ (For cell 10 and machine type 13)

$Y_1^{11} + Y_{14}^{11} \le 1$ (For cell 11 and machine type 1)
$Y_2^{11} + Y_{15}^{11} \le 1$ (For cell 11 and machine type 2)
$Y_3^{11} \le 1$ (For cell 11 and machine type 3)
$Y_4^{11} \le 1$ (For cell 11 and machine type 4)
$Y_5^{11} \le 1$ (For cell 11 and machine type 5)
$Y_6^{11} \le 1$ (For cell 11 and machine type 6)
$Y_7^{11} \le 1$ (For cell 11 and machine type 7)
$Y_8^{11} \le 1$ (For cell 11 and machine type 8)
$Y_9^{11} \le 1$ (For cell 11 and machine type 9)
$Y_{10}^{11} \le 1$ (For cell 11 and machine type 10)
$Y_{11}^{11} \le 1$ (For cell 11 and machine type 11)
$Y_{12}^{11} \le 1$ (For cell 11 and machine type 12)
$Y_{13}^{11} \le 1$ (For cell 11 and machine type 13)

$Y_1^{12} + Y_{14}^{12} \le 1$ (For cell 12 and machine type 1)
$Y_2^{12} + Y_{15}^{12} \le 1$ (For cell 12 and machine type 2)
$Y_3^{12} \le 1$ (For cell 12 and machine type 3)
$Y_4^{12} \le 1$ (For cell 12 and machine type 4)
$Y_5^{12} \le 1$ (For cell 12 and machine type 5)
$Y_6^{12} \le 1$ (For cell 12 and machine type 6)
$Y_7^{12} \le 1$ (For cell 12 and machine type 7)
$Y_8^{12} \le 1$ (For cell 12 and machine type 8)
$Y_9^{12} \le 1$ (For cell 12 and machine type 9)
$Y_{10}^{12} \le 1$ (For cell 12 and machine type 10)
$Y_{11}^{12} \le 1$ (For cell 12 and machine type 11)
$Y_{12}^{12} \le 1$ (For cell 12 and machine type 12)
$Y_{13}^{12} \le 1$ (For cell 12 and machine type 13)

$Y_1^{13} + Y_{14}^{13} \le 1$ (For cell 13 and machine type 1)
$Y_2^{13} + Y_{15}^{13} \le 1$ (For cell 13 and machine type 2)
$Y_3^{13} \le 1$ (For cell 13 and machine type 3)
$Y_4^{13} \le 1$ (For cell 13 and machine type 4)
$Y_5^{13} \le 1$ (For cell 13 and machine type 5)
$Y_6^{13} \le 1$ (For cell 13 and machine type 6)
$Y_7^{13} \le 1$ (For cell 13 and machine type 7)
$Y_8^{13} \le 1$ (For cell 13 and machine type 8)
$Y_9^{13} \le 1$ (For cell 13 and machine type 9)
$Y_{10}^{13} \le 1$ (For cell 13 and machine type 10)
$Y_{11}^{13} \le 1$ (For cell 13 and machine type 11)
$Y_{12}^{13} \le 1$ (For cell 13 and machine type 12)
$Y_{13}^{13} \le 1$ (For cell 13 and machine type 13)

$Y_1^{14} + Y_{14}^{14} \le 1$ (For cell 14 and machine type 1)
$Y_2^{14} + Y_{15}^{14} \le 1$ (For cell 14 and machine type 2)
$Y_3^{14} \le 1$ (For cell 14 and machine type 3)
$Y_4^{14} \le 1$ (For cell 14 and machine type 4)
$Y_5^{14} \le 1$ (For cell 14 and machine type 5)
$Y_6^{14} \le 1$ (For cell 14 and machine type 6)
$Y_7^{14} \le 1$ (For cell 14 and machine type 7)
$Y_8^{14} \le 1$ (For cell 14 and machine type 8)
$Y_9^{14} \le 1$ (For cell 14 and machine type 9)
$Y_{10}^{14} \le 1$ (For cell 14 and machine type 10)
$Y_{11}^{14} \le 1$ (For cell 14 and machine type 11)
$Y_{12}^{14} \le 1$ (For cell 14 and machine type 12)
$Y_{13}^{14} \le 1$ (For cell 14 and machine type 13)

$Y_1^{15} + Y_{14}^{15} \le 1$ (For cell 15 and machine type 1)
$Y_2^{15} + Y_{15}^{15} \le 1$ (For cell 15 and machine type 2)
$Y_3^{15} \le 1$ (For cell 15 and machine type 3)
$Y_4^{15} \le 1$ (For cell 15 and machine type 4)
$Y_5^{15} \le 1$ (For cell 15 and machine type 5)
$Y_6^{15} \le 1$ (For cell 15 and machine type 6)
$Y_7^{15} \le 1$ (For cell 15 and machine type 7)
$Y_8^{15} \le 1$ (For cell 15 and machine type 8)
$Y_9^{15} \le 1$ (For cell 15 and machine type 9)
$Y_{10}^{15} \le 1$ (For cell 15 and machine type 10)
$Y_{11}^{15} \le 1$ (For cell 15 and machine type 11)
$Y_{12}^{15} \le 1$ (For cell 15 and machine type 12)
$Y_{13}^{15} \le 1$ (For cell 15 and machine type 13)

(f) Machines type availability constraint

$$Y_1^1 + Y_1^2 + Y_1^3 + Y_1^4 + Y_1^5 + Y_1^6 + Y_1^7 + Y_1^8 + Y_1^9 + Y_1^{10} + Y_1^{11} + Y_1^{12} + Y_1^{13} + Y_1^{14} + Y_1^{15} +$$
$$Y_{14}^1 + Y_{14}^2 + Y_{14}^3 + Y_{14}^4 + Y_{14}^5 + Y_{14}^6 + Y_{14}^7 + Y_{14}^8 + Y_{14}^9 + Y_{14}^{10} + Y_{14}^{11} + Y_{14}^{12} + Y_{14}^{13} + Y_{14}^{14} + Y_{14}^{15} = 2 \text{ ; (For machine type 1)}$$
$$Y_2^1 + Y_2^2 + Y_2^3 + Y_2^4 + Y_2^5 + Y_2^6 + Y_2^7 + Y_2^8 + Y_2^9 + Y_2^{10} + Y_2^{11} + Y_2^{12} + Y_2^{13} + Y_2^{14} + Y_2^{15} +$$
$$Y_{15}^1 + Y_{15}^2 + Y_{15}^3 + Y_{15}^4 + Y_{15}^5 + Y_{15}^6 + Y_{15}^7 + Y_{15}^8 + Y_{15}^9 + Y_{15}^{10} + Y_{15}^{11} + Y_{15}^{12} + Y_{15}^{13} + Y_{15}^{14} + Y_{15}^{15} = 2 \text{ ; (For machine type 2)}$$
$$Y_3^1 + Y_3^2 + Y_3^3 + Y_3^4 + Y_3^5 + Y_3^6 + Y_3^7 + Y_3^8 + Y_3^9 + Y_3^{10} + Y_3^{11} + Y_3^{12} + Y_3^{13} + Y_3^{14} + Y_3^{15} = 1 \text{ ; (For machine type 3)}$$
$$Y_4^1 + Y_4^2 + Y_4^3 + Y_4^4 + Y_4^5 + Y_4^6 + Y_4^7 + Y_4^8 + Y_4^9 + Y_4^{10} + Y_4^{11} + Y_4^{12} + Y_4^{13} + Y_4^{14} + Y_4^{15} = 1 \text{ ; (For machine type 4)}$$
$$Y_5^1 + Y_5^2 + Y_5^3 + Y_5^4 + Y_5^5 + Y_5^6 + Y_5^7 + Y_5^8 + Y_5^9 + Y_5^{10} + Y_5^{11} + Y_5^{12} + Y_5^{13} + Y_5^{14} + Y_5^{15} = 1 \text{ ; (For machine type 5)}$$
$$Y_6^1 + Y_6^2 + Y_6^3 + Y_6^4 + Y_6^5 + Y_6^6 + Y_6^7 + Y_6^8 + Y_6^9 + Y_6^{10} + Y_6^{11} + Y_6^{12} + Y_6^{13} + Y_6^{14} + Y_6^{15} = 1 \text{ ; (For machine type 6)}$$
$$Y_7^1 + Y_7^2 + Y_7^3 + Y_7^4 + Y_7^5 + Y_7^6 + Y_7^7 + Y_7^8 + Y_7^9 + Y_7^{10} + Y_7^{11} + Y_7^{12} + Y_7^{13} + Y_7^{14} + Y_7^{15} = 1 \text{ ; (For machine type 7)}$$
$$Y_8^1 + Y_8^2 + Y_8^3 + Y_8^4 + Y_8^5 + Y_8^6 + Y_8^7 + Y_8^8 + Y_8^9 + Y_8^{10} + Y_8^{11} + Y_8^{12} + Y_8^{13} + Y_8^{14} + Y_8^{15} = 1 \text{ ; (For machine type 8)}$$
$$Y_9^1 + Y_9^2 + Y_9^3 + Y_9^4 + Y_9^5 + Y_9^6 + Y_9^7 + Y_9^8 + Y_9^9 + Y_9^{10} + Y_9^{11} + Y_9^{12} + Y_9^{13} + Y_9^{14} + Y_9^{15} = 1 \text{ ; (For machine type 9)}$$
$$Y_{10}^1 + Y_{10}^2 + Y_{10}^3 + Y_{10}^4 + Y_{10}^5 + Y_{10}^6 + Y_{10}^7 + Y_{10}^8 + Y_{10}^9 + Y_{10}^{10} + Y_{10}^{11} + Y_{10}^{12} + Y_{10}^{13} + Y_{10}^{14} + Y_{10}^{15} = 1 \text{ ; (For machine type 10)}$$
$$Y_{11}^1 + Y_{11}^2 + Y_{11}^3 + Y_{11}^4 + Y_{11}^5 + Y_{11}^6 + Y_{11}^7 + Y_{11}^8 + Y_{11}^9 + Y_{11}^{10} + Y_{11}^{11} + Y_{11}^{12} + Y_{11}^{13} + Y_{11}^{14} + Y_{11}^{15} = 1 \text{ ; (For machine type 11)}$$
$$Y_{12}^1 + Y_{12}^2 + Y_{12}^3 + Y_{12}^4 + Y_{12}^5 + Y_{12}^6 + Y_{12}^7 + Y_{12}^8 + Y_{12}^9 + Y_{12}^{10} + Y_{12}^{11} + Y_{12}^{12} + Y_{12}^{13} + Y_{12}^{14} + Y_{12}^{15} = 1 \text{ ; (For machine type 12)}$$
$$Y_{13}^1 + Y_{13}^2 + Y_{13}^3 + Y_{13}^4 + Y_{13}^5 + Y_{13}^6 + Y_{13}^7 + Y_{13}^8 + Y_{13}^9 + Y_{13}^{10} + Y_{13}^{11} + Y_{13}^{12} + Y_{13}^{13} + Y_{13}^{14} + Y_{13}^{15} = 1 \text{ ; (For machine type 13)}$$

(g) Operation feasiblity constraint

$$X_{p,m}^{r,o} \le A_{p,m}^{r,o} \qquad p = 1, \ldots, 10; \qquad r = 1, 2;$$
$$o = 1, \ldots, O_p^r; \qquad m = 1, \ldots, 15.$$

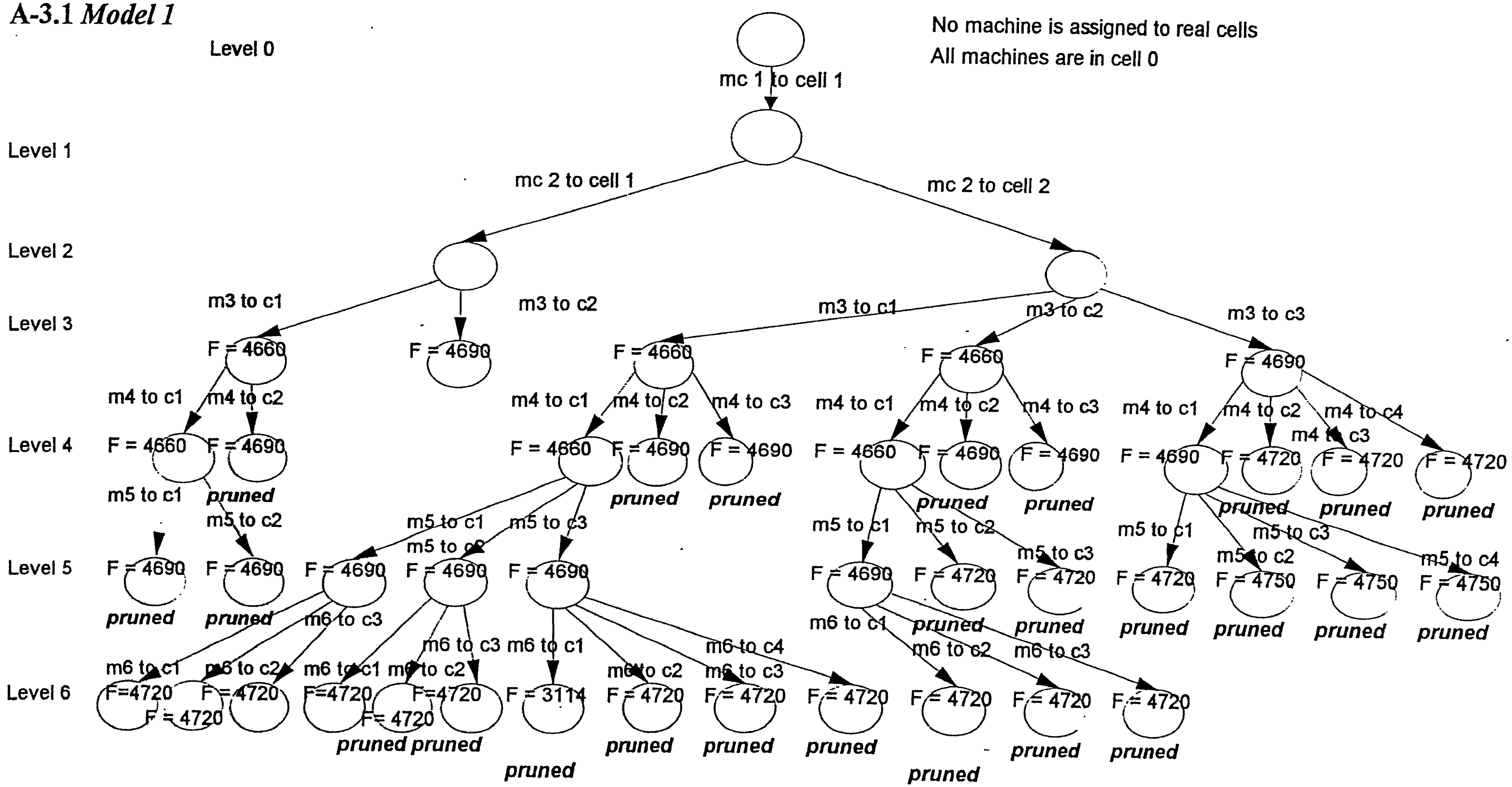
Decision Variables

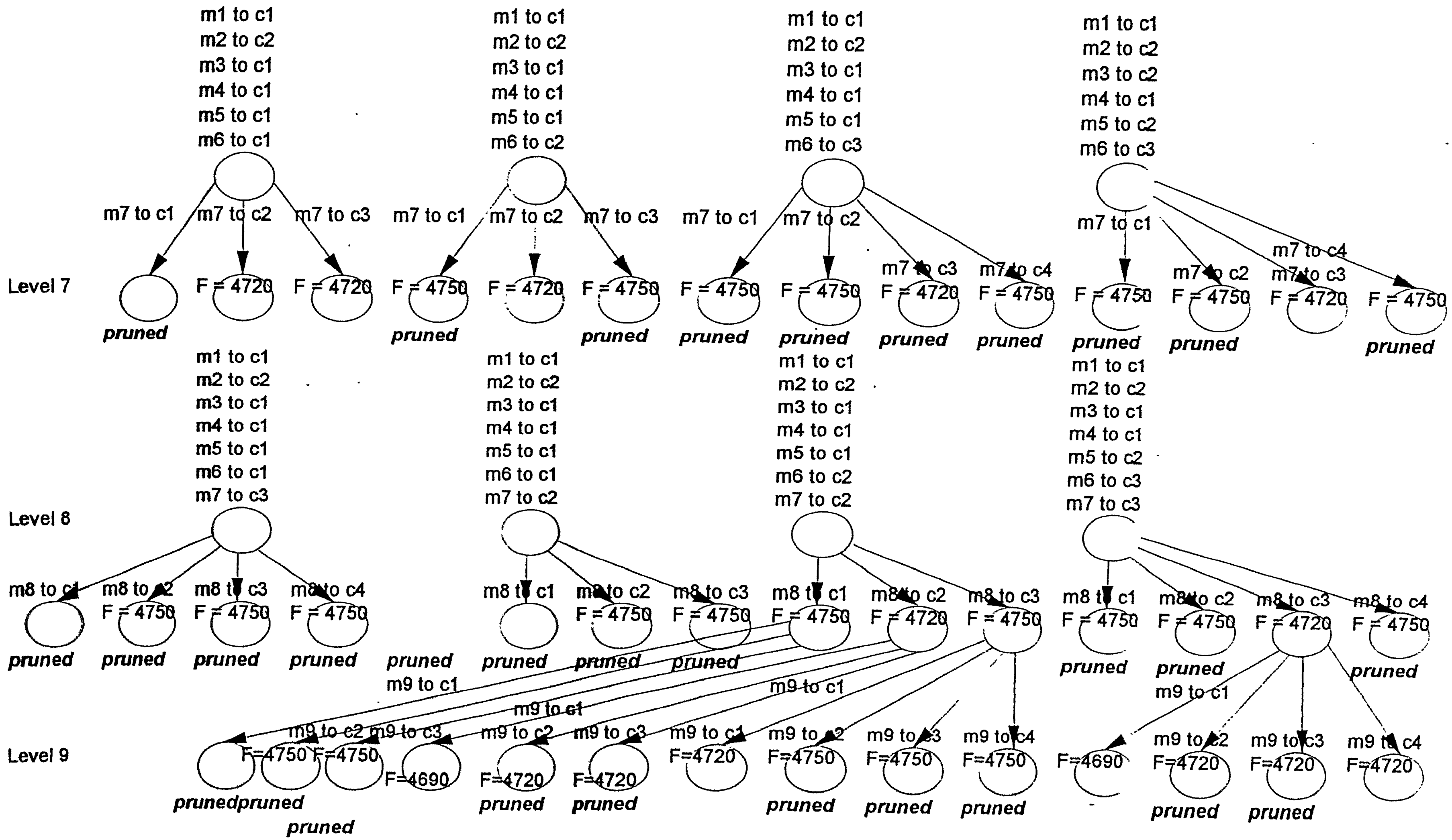
$$Y_m^c \in \{0,1\} \qquad m = 1, \ldots, 15; \qquad c = 1, \ldots, 15.$$

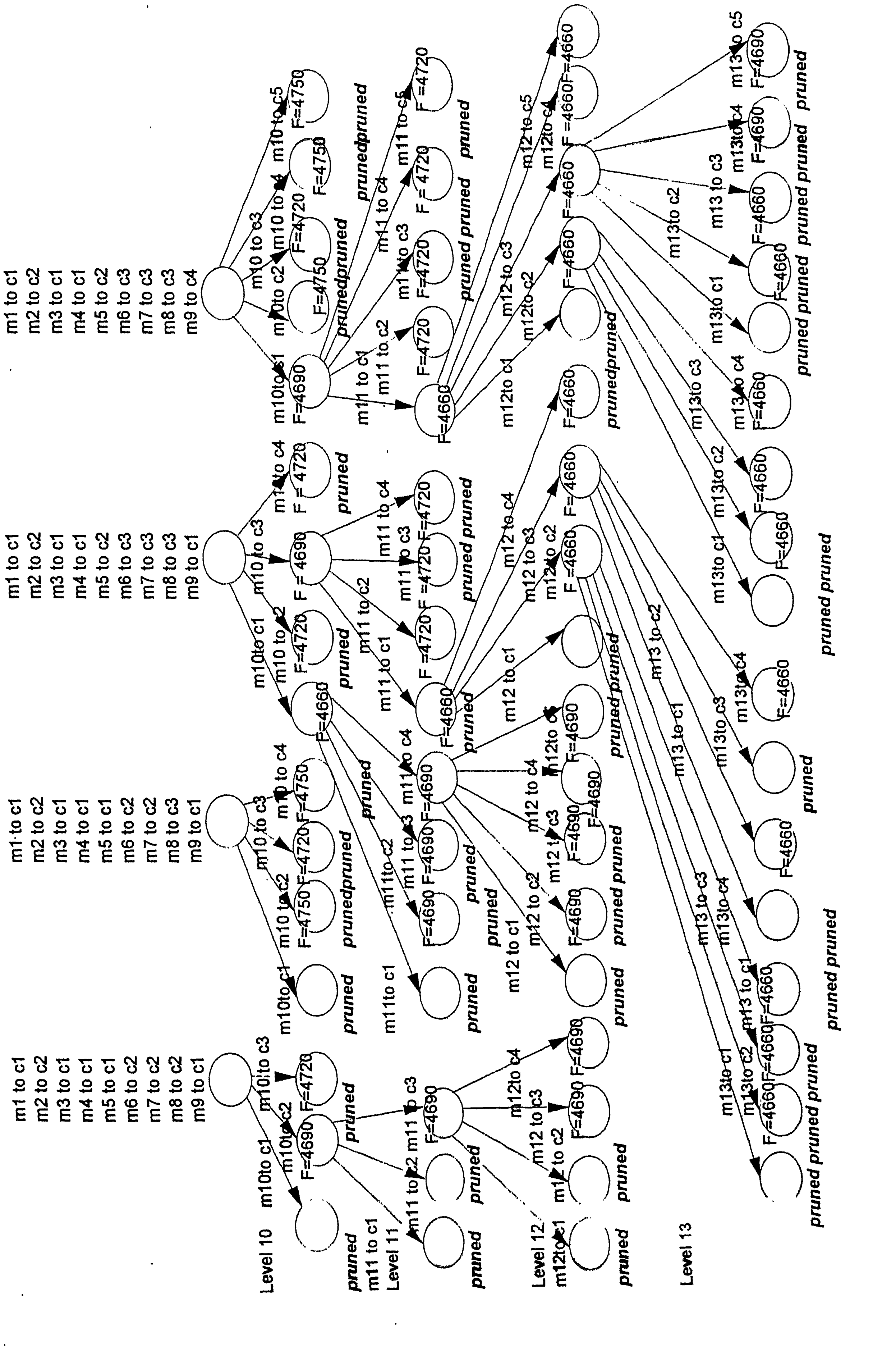
$$Z_p^r \in \{0,1\} \qquad p = 1, \ldots, 10; \qquad r = 1, 2.$$

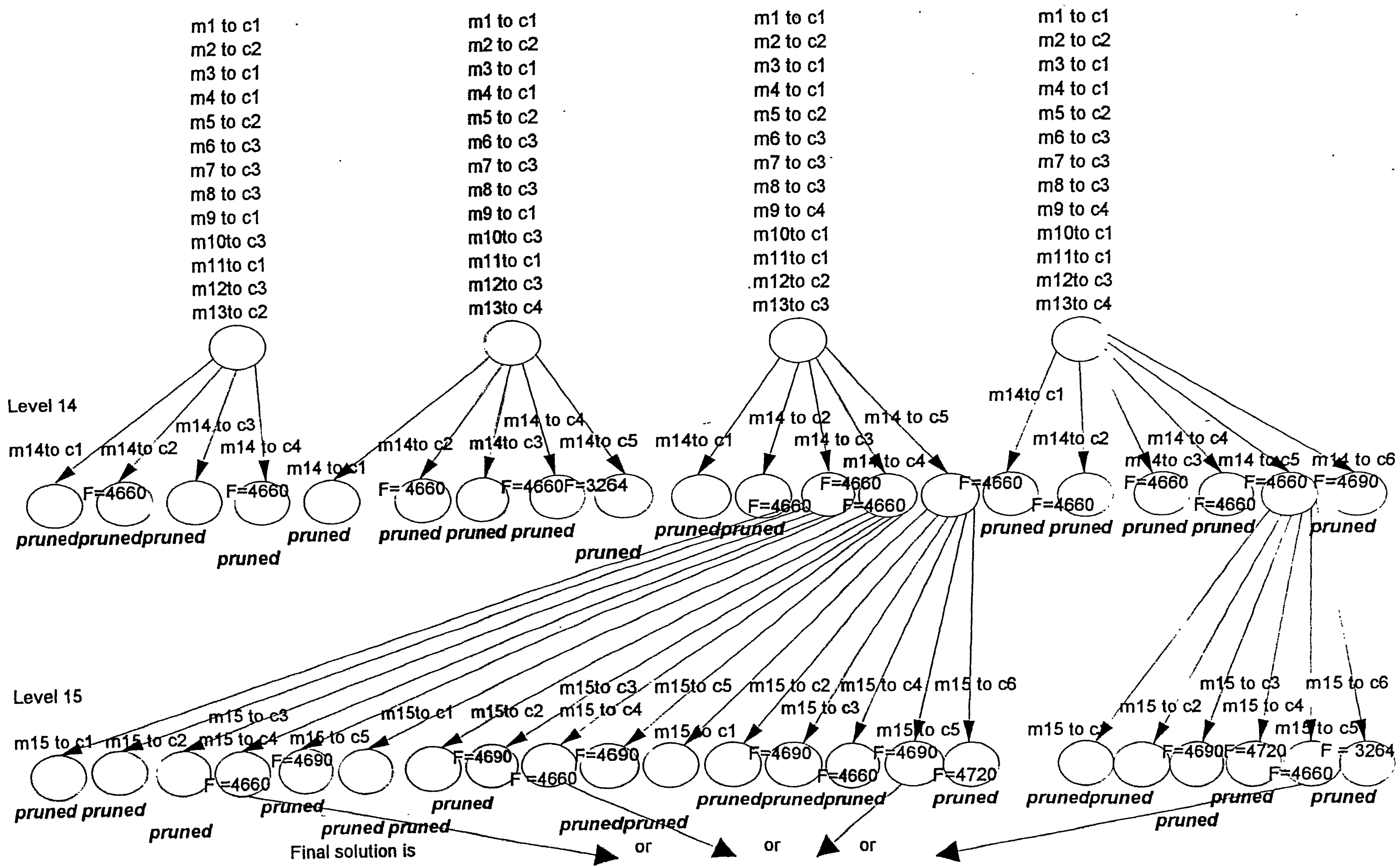
A-3 Illustration of Solution Methodology

A-3.1 *Model 1*

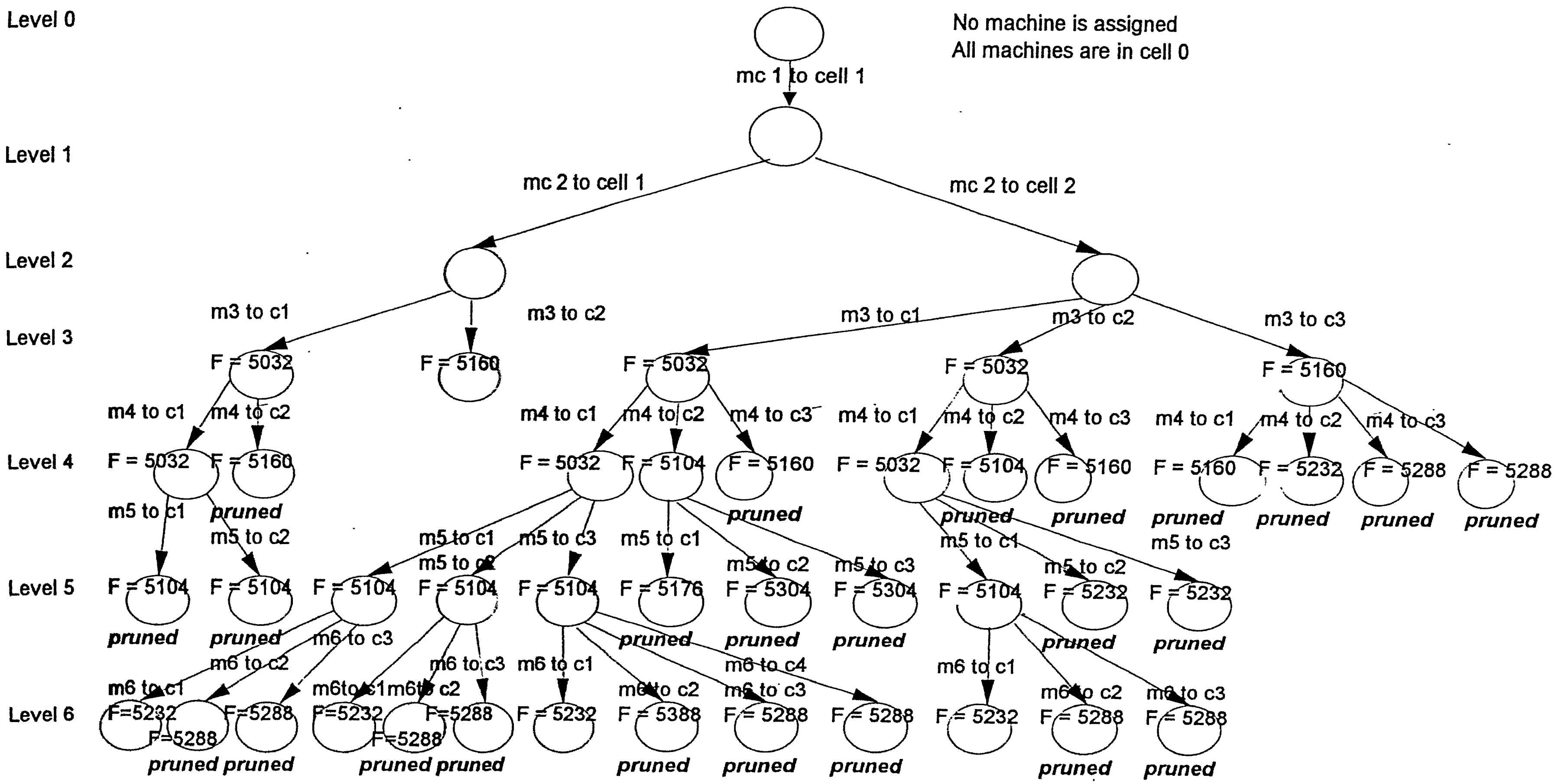



Level 7
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c1
m7 to c1
m7 to c2
m7 to c3
F = 4720
F = 4720
pruned
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c2
m7 to c1
m7 to c2
m7 to c3
F = 4750
F = 4720
F = 4750
pruned
pruned
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c3
m7 to c1
m7 to c2
m7 to c3
m7 to c4
F = 4750
F = 4750
F = 4720
F = 4750
pruned
pruned
pruned
pruned
m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c2
m6 to c3
m7 to c1
m7 to c2
m7 to c3
m7 to c4
F = 4750
F = 4750
F = 4720
F = 4750
pruned
pruned
pruned
Level 8
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c1
m7 to c3
m8 to c1
m8 to c2
m8 to c3
m8 to c4
F = 4750
F = 4750
F = 4750
pruned
pruned
pruned
pruned
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c1
m7 to c2
m8 to c1
m8 to c2
m8 to c3
F = 4750
F = 4750
pruned
pruned
pruned
pruned
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c2
m7 to c2
m8 to c1
m8 to c2
m8 to c3
F = 4750
F = 4720
F = 4750
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c2
m6 to c3
m7 to c3
m8 to c1
m8 to c2
m8 to c3
m8 to c4
F = 4750
F = 4750
F = 4720
F = 4750
pruned
pruned
pruned
Level 9
m9 to c1
m9 to c2
m9 to c3
m9 to c1
m9 to c2
m9 to c3
m9 to c1
m9 to c1
m9 to c2
m9 to c3
m9 to c4
F=4750
F=4750
F=4690
F=4720
F=4720
F=4720
F=4750
F=4750
F=4750
prunedpruned
pruned
pruned
pruned
pruned
pruned
pruned
m9 to c1
m9 to c2
m9 to c3
m9 to c4
F=4690
F=4720
F=4720
F=4720
pruned
pruned

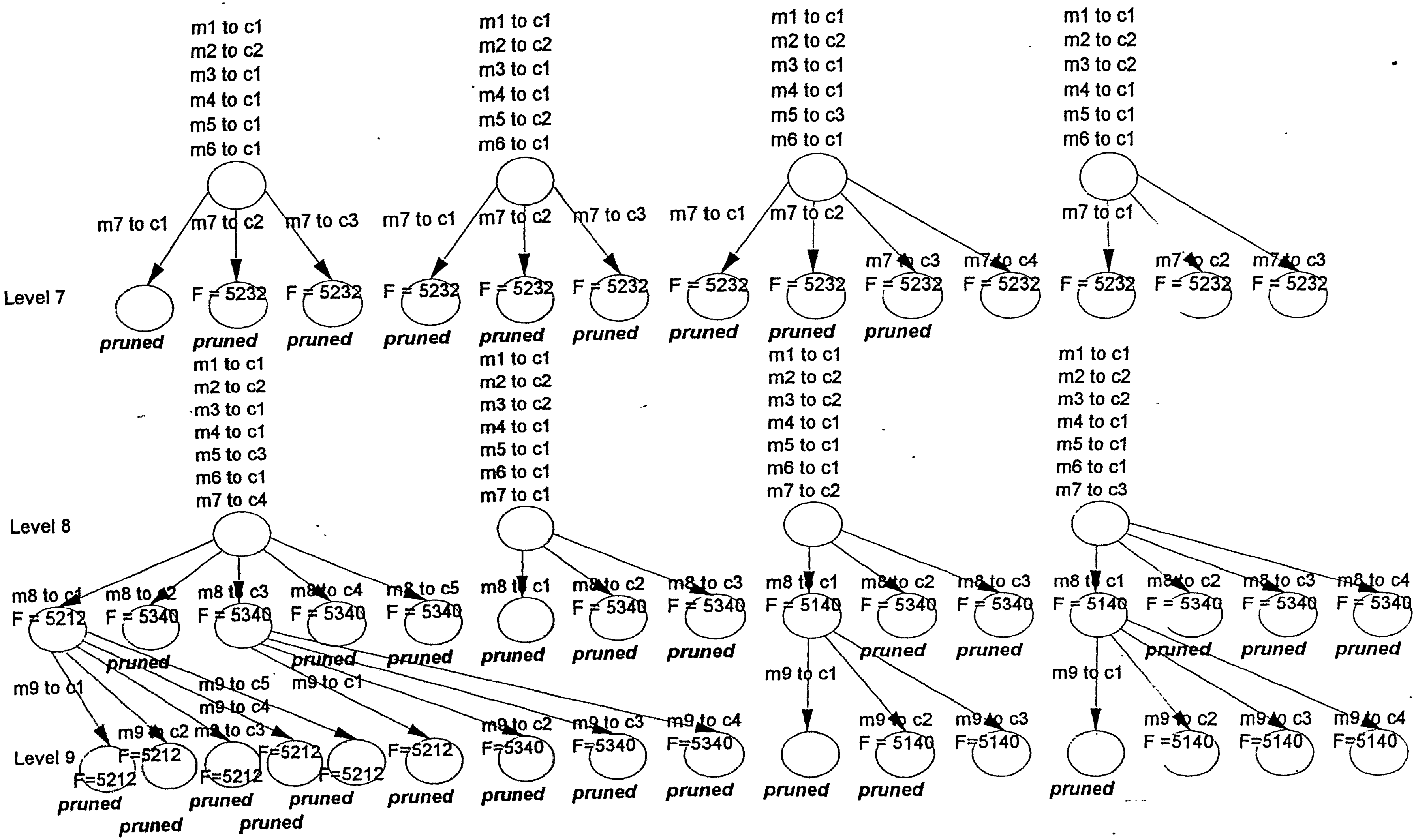

Level 10
Level 11
Level 12
Level 13
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c2
m7 to c2
m8 to c2
m9 to c1
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c2
m7 to c2
m8 to c3
m9 to c1
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c2
m6 to c3
m7 to c3
m8 to c3
m9 to c1
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c2
m6 to c3
m7 to c3
m8 to c3
m9 to c4
pruned


m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c2
m6 to c3
m7 to c3
m8 to c3
m9 to c1
m10to c3
m11to c1
m12to c3
m13to c2
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c2
m6 to c3
m7 to c3
m8 to c3
m9 to c1
m10to c3
m11to c1
m12to c3
m13to c4
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c2
m6 to c3
m7 to c3
m8 to c3
m9 to c4
m10to c1
m11to c1
m12to c2
m13to c3
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c2
m6 to c3
m7 to c3
m8 to c3
m9 to c4
m10to c1
m11to c1
m12to c3
m13to c4
Level 14
Level 15
Final solution is
or
or
or

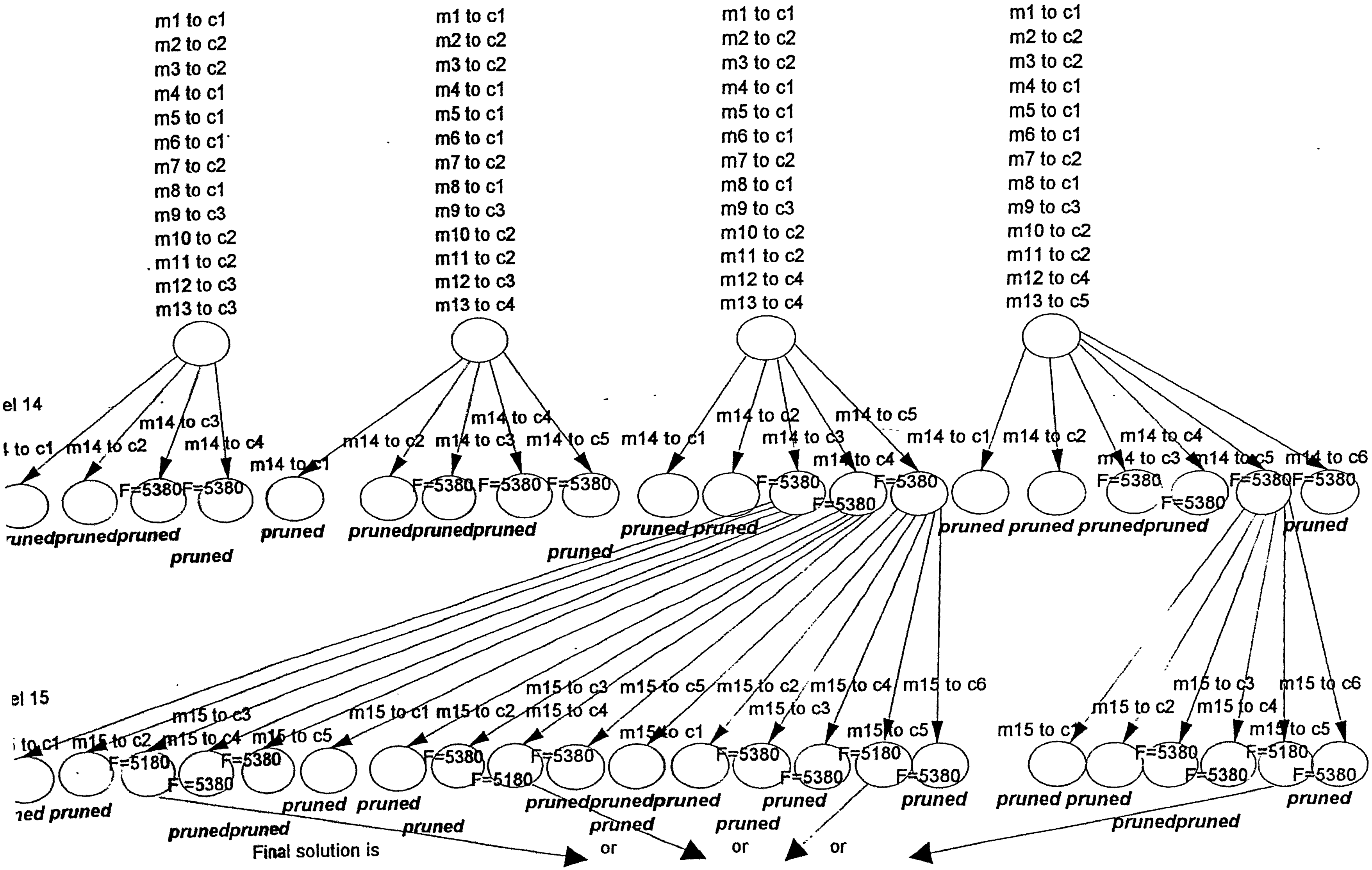
A-3.2 *Model 2 :*



Level 0
Level 1
Level 2
Level 3
Level 4
Level 5
Level 6
No machine is assigned
All machines are in cell 0
mc 1 to cell 1
mc 2 to cell 1
mc 2 to cell 2
m3 to c1
m3 to c2
m3 to c3
m4 to c1
m4 to c2
m4 to c3
m5 to c1
m5 to c2
m5 to c3
m6 to c1
m6 to c2
m6 to c3
m6 to c4
F = 5032
F = 5104
F = 5160
F = 5176
F = 5232
F = 5288
F = 5304
F = 5388
pruned


Level 7
Level 8
Level 9
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c1
m6 to c1
m7 to c1
m7 to c2
m7 to c3
F = 5232
pruned
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c2
m6 to c1
m1 to c1
m2 to c2
m3 to c1
m4 to c1
m5 to c3
m6 to c1
m7 to c4
m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c1
m7 to c2
m7 to c3
m8 to c1
m8 to c2
m8 to c3
m8 to c4
m8 to c5
F = 5212
F = 5340
F = 5140
m9 to c1
m9 to c2
m9 to c3
m9 to c4
m9 to c5
F=5212
F=5340
F=5140


m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c2
m8 to c1
m9 to c3
m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c3
m8 to c1
m9 to c2
m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c3
m8 to c1
m9 to c3
m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c3
m8 to c1
m9 to c4
Level 10
Level 11
Level 12
Level 13
pruned
F=5308
F=5436
F=5180
F=5236



m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c2
m8 to c1
m9 to c3
m10 to c2
m11 to c2
m12 to c3
m13 to c3
m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c2
m8 to c1
m9 to c3
m10 to c2
m11 to c2
m12 to c3
m13 to c4
m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c2
m8 to c1
m9 to c3
m10 to c2
m11 to c2
m12 to c4
m13 to c4
m1 to c1
m2 to c2
m3 to c2
m4 to c1
m5 to c1
m6 to c1
m7 to c2
m8 to c1
m9 to c3
m10 to c2
m11 to c2
m12 to c4
m13 to c5
el 14
el 15
m14 to c1
m14 to c2
m14 to c3
m14 to c4
m14 to c5
m14 to c6
m15 to c1
m15 to c2
m15 to c3
m15 to c4
m15 to c5
m15 to c6
F=5380
F=5180
pruned
Final solution is
or
or
or

A-4 Solution(s)

A-4.1 *Model 1*

$Y_1^1=1;\ Y_2^2=1;\ Y_3^1=1;\ Y_4^1=1;\ Y_5^2=1;\ Y_6^3=1;\ Y_7^3=1;\ Y_8^3=1;\ Y_9^4=1;\ Y_{10}^1=1;\ Y_{11}^1=1;\ Y_{12}^2=1;$
$Y_{13}^3=1;\ Y_{14}^3=1;\ Y_{15}^3=1;$
$Z_1^1=1;\ Z_2^1=1;\ Z_3^2=1;\ Z_4^1 or Z_4^2=1;\ Z_5^1=1;\ Z_6^1 or Z_6^2=1;\ Z_7^2=1;\ Z_8^1=1;\ Z_9^1=1;\ Z_{10}^1=1;$
$X_{1,1}^{1,1}=1; X_{1,3}^{1,2}=1; X_{1,10}^{1,3}=1; X_{1,2}^{2,1}=1; X_{1,3}^{2,2}=1; X_{1,13}^{2,3}=1; X_{2,10}^{1,1}=1; X_{2,8}^{1,2}=1; X_{2,15}^{2,1}=1; X_{2,14}^{2,2}=1; X_{2,10}^{2,3}=1;$
$X_{3,2}^{1,1}=1; X_{3,10}^{1,2}=1; X_{3,1}^{2,1}=1; X_{3,11}^{2,2}=1; X_{4,3}^{1,1}=1; X_{4,1}^{1,2}=1; X_{4,14}^{2,1}=1; X_{4,15}^{2,2}=1; X_{5,3}^{1,1}=1; X_{5,11}^{1,2}=1; X_{5,2}^{2,1}=1; X_{5,10}^{2,2}=1;$
$X_{5,11}^{2,3}=1; X_{6,5}^{1,1}=1; X_{6,1}^{1,2}=1; X_{6,4}^{2,1}=1; X_{6,3}^{2,2}=1; X_{7,2}^{1,1}=1; X_{7,4}^{1,2}=1; X_{7,11}^{1,3}=1; X_{7,4}^{2,1}=1; X_{7,1}^{2,2}=1; X_{7,15}^{2,3}=1; X_{8,10}^{1,1}=1;$
$X_{8,14}^{1,2}=1; X_{8,2}^{2,1}=1; X_{8,10}^{2,2}=1; X_{8,14}^{2,3}=1; X_{9,6}^{1,1}=1; X_{9,7}^{1,2}=1; X_{9,8}^{1,3}=1; X_{9,1}^{2,1}=1; X_{9,6}^{2,2}=1; X_{9,8}^{2,3}=1; X_{10,9}^{1,1}=1; X_{10,4}^{1,2}=1;$
$X_{10,1}^{1,3}=1; X_{10,8}^{2,1}=1; X_{10,6}^{2,2}=1; X_{10,5}^{2,3}$

or

$Y_1^1=1;\ Y_2^2=1;\ Y_3^1=1;\ Y_4^1=1;\ Y_5^2=1;\ Y_6^3=1;\ Y_7^3=1;\ Y_8^3=1;\ Y_9^4=1;\ Y_{10}^1=1;\ Y_{11}^1=1;\ Y_{12}^2=1;$
$Y_{13}^3=1;\ Y_{14}^4=1;\ Y_{15}^4=1;$
$Z_1^1=1;\ Z_2^1=1;\ Z_3^2=1;\ Z_4^1 or Z_4^2=1;\ Z_5^1=1;\ Z_6^1 or Z_6^2=1;\ Z_7^2=1;\ Z_8^1=1;\ Z_9^1=1;\ Z_{10}^1=1;$
$X_{1,1}^{1,1}=1; X_{1,3}^{1,2}=1; X_{1,10}^{1,3}=1; X_{1,2}^{2,1}=1; X_{1,3}^{2,2}=1; X_{1,13}^{2,3}=1; X_{2,10}^{1,1}=1; X_{2,8}^{1,2}=1; X_{2,15}^{2,1}=1; X_{2,14}^{2,2}=1; X_{2,10}^{2,3}=1;$
$X_{3,2}^{1,1}=1; X_{3,10}^{1,2}=1; X_{3,1}^{2,1}=1; X_{3,11}^{2,2}=1; X_{4,3}^{1,1}=1; X_{4,1}^{1,2}=1; X_{4,14}^{2,1}=1; X_{4,15}^{2,2}=1; X_{5,3}^{1,1}=1; X_{5,11}^{1,2}=1; X_{5,2}^{2,1}=1; X_{5,10}^{2,2}=1;$
$X_{5,11}^{2,3}=1; X_{6,5}^{1,1}=1; X_{6,1}^{1,2}=1; X_{6,4}^{2,1}=1; X_{6,3}^{2,2}=1; X_{7,2}^{1,1}=1; X_{7,4}^{1,2}=1; X_{7,11}^{1,3}=1; X_{7,4}^{2,1}=1; X_{7,1}^{2,2}=1; X_{7,15}^{2,3}=1; X_{8,10}^{1,1}=1;$
$X_{8,14}^{1,2}=1; X_{8,2}^{2,1}=1; X_{8,10}^{2,2}=1; X_{8,14}^{2,3}=1; X_{9,6}^{1,1}=1; X_{9,7}^{1,2}=1; X_{9,8}^{1,3}=1; X_{9,1}^{2,1}=1; X_{9,6}^{2,2}=1; X_{9,8}^{2,3}=1; X_{10,9}^{1,1}=1; X_{10,4}^{1,2}=1;$
$X_{10,1}^{1,3}=1; X_{10,8}^{2,1}=1; X_{10,6}^{2,2}=1; X_{10,5}^{2,3}$

or

$Y_1^1=1;\ Y_2^2=1;\ Y_3^1=1;\ Y_4^1=1;\ Y_5^2=1;\ Y_6^3=1;\ Y_7^3=1;\ Y_8^3=1;\ Y_9^4=1;\ Y_{10}^1=1;\ Y_{11}^1=1;\ Y_{12}^2=1;$
$Y_{13}^3=1;\ Y_{14}^5=1;\ Y_{15}^5=1;$
$Z_1^1=1;\ Z_2^1=1;\ Z_3^2=1;\ Z_4^1 or Z_4^2=1;\ Z_5^1=1;\ Z_6^1 or Z_6^2=1;\ Z_7^2=1;\ Z_8^1=1;\ Z_9^1=1;\ Z_{10}^1=1;$
$X_{1,1}^{1,1}=1; X_{1,3}^{1,2}=1; X_{1,10}^{1,3}=1; X_{1,2}^{2,1}=1; X_{1,3}^{2,2}=1; X_{1,13}^{2,3}=1; X_{2,10}^{1,1}=1; X_{2,8}^{1,2}=1; X_{2,15}^{2,1}=1; X_{2,14}^{2,2}=1; X_{2,10}^{2,3}=1;$
$X_{3,2}^{1,1}=1; X_{3,10}^{1,2}=1; X_{3,1}^{2,1}=1; X_{3,11}^{2,2}=1; X_{4,3}^{1,1}=1; X_{4,1}^{1,2}=1; X_{4,14}^{2,1}=1; X_{4,15}^{2,2}=1; X_{5,3}^{1,1}=1; X_{5,11}^{1,2}=1; X_{5,2}^{2,1}=1; X_{5,10}^{2,2}=1;$
$X_{5,11}^{2,3}=1; X_{6,5}^{1,1}=1; X_{6,1}^{1,2}=1; X_{6,4}^{2,1}=1; X_{6,3}^{2,2}=1; X_{7,2}^{1,1}=1; X_{7,4}^{1,2}=1; X_{7,11}^{1,3}=1; X_{7,4}^{2,1}=1; X_{7,1}^{2,2}=1; X_{7,15}^{2,3}=1; X_{8,10}^{1,1}=1;$
$X_{8,14}^{1,2}=1; X_{8,2}^{2,1}=1; X_{8,10}^{2,2}=1; X_{8,14}^{2,3}=1; X_{9,6}^{1,1}=1; X_{9,7}^{1,2}=1; X_{9,8}^{1,3}=1; X_{9,1}^{2,1}=1; X_{9,6}^{2,2}=1; X_{9,8}^{2,3}=1; X_{10,9}^{1,1}=1; X_{10,4}^{1,2}=1;$
$X_{10,1}^{1,3}=1; X_{10,8}^{2,1}=1; X_{10,6}^{2,2}=1; X_{10,5}^{2,3}$

or

$Y_1^1=1;\ Y_2^2=1;\ Y_3^1=1;\ Y_4^1=1;\ Y_5^2=1;\ Y_6^3=1;\ Y_7^3=1;\ Y_8^3=1;\ Y_9^4=1;\ Y_{10}^1=1;\ Y_{11}^1=1;\ Y_{12}^3=1;$
$Y_{13}^4=1;\ Y_{14}^5=1;\ Y_{15}^5=1;$
$Z_1^1=1;\ Z_2^1=1;\ Z_3^2=1;\ Z_4^1 or Z_4^2=1;\ Z_5^1=1;\ Z_6^1 or Z_6^2=1;\ Z_7^2=1;\ Z_8^1=1;\ Z_9^1=1;\ Z_{10}^1=1;$
$X_{1,1}^{1,1}=1; X_{1,3}^{1,2}=1; X_{1,10}^{1,3}=1; X_{1,2}^{2,1}=1; X_{1,3}^{2,2}=1; X_{1,13}^{2,3}=1; X_{2,10}^{1,1}=1; X_{2,8}^{1,2}=1; X_{2,15}^{2,1}=1; X_{2,14}^{2,2}=1; X_{2,10}^{2,3}=1;$
$X_{3,2}^{1,1}=1; X_{3,10}^{1,2}=1; X_{3,1}^{2,1}=1; X_{3,11}^{2,2}=1; X_{4,3}^{1,1}=1; X_{4,1}^{1,2}=1; X_{4,14}^{2,1}=1; X_{4,15}^{2,2}=1; X_{5,3}^{1,1}=1; X_{5,11}^{1,2}=1; X_{5,2}^{2,1}=1; X_{5,10}^{2,2}=1;$
$X_{5,11}^{2,3}=1; X_{6,5}^{1,1}=1; X_{6,1}^{1,2}=1; X_{6,4}^{2,1}=1; X_{6,3}^{2,2}=1; X_{7,2}^{1,1}=1; X_{7,4}^{1,2}=1; X_{7,11}^{1,3}=1; X_{7,4}^{2,1}=1; X_{7,1}^{2,2}=1; X_{7,15}^{2,3}=1; X_{8,10}^{1,1}=1;$
$X_{8,14}^{1,2}=1; X_{8,2}^{2,1}=1; X_{8,10}^{2,2}=1; X_{8,14}^{2,3}=1; X_{9,6}^{1,1}=1; X_{9,7}^{1,2}=1; X_{9,8}^{1,3}=1; X_{9,1}^{2,1}=1; X_{9,6}^{2,2}=1; X_{9,8}^{2,3}=1; X_{10,9}^{1,1}=1; X_{10,4}^{1,2}=1;$
$X_{10,1}^{1,3}=1; X_{10,8}^{2,1}=1; X_{10,6}^{2,2}=1; X_{10,5}^{2,3}$

Objective Function $F=4660$

A-4.2 *Model 2*

$Y_1^1 = 1;\ Y_2^2 = 1;\ Y_3^2 = 1;\ Y_4^1 = 1;\ Y_5^1 = 1;\ Y_6^1 = 1;\ Y_7^2 = 1;\ Y_8^1 = 1;\ Y_9^3 = 1;\ Y_{10}^2 = 1;\ Y_{11}^2 = 1;\ Y_{12}^4 = 1;$
$Y_{13}^4 = 1;\ Y_{14}^3 = 1;\ Y_{15}^3 = 1;$
$Z_1^2 = 1;\ Z_2^1 = 1;\ Z_3^1 = 1;\ Z_4^2 = 1;\ Z_5^1 = 1;\ Z_6^1 = 1;\ Z_7^2 = 1;\ Z_8^1 = 1;\ Z_9^2 = 1;\ Z_{10}^2 = 1;$
$X_{1,1}^{1,1} = 1; X_{1,3}^{1,2} = 1; X_{1,10}^{1,3} = 1; X_{1,2}^{2,1} = 1; X_{1,3}^{2,2} = 1; X_{1,13}^{2,3} = 1; X_{2,10}^{1,1} = 1; X_{2,8}^{1,2} = 1; X_{2,15}^{2,1} = 1; X_{2,14}^{2,2} = 1; X_{2,10}^{2,3} = 1;$
$X_{3,2}^{1,1} = 1; X_{3,10}^{1,2} = 1; X_{3,1}^{2,1} = 1; X_{3,11}^{2,2} = 1; X_{4,3}^{1,1} = 1; X_{4,1}^{1,2} = 1; X_{4,14}^{2,1} = 1; X_{4,15}^{2,2} = 1; X_{5,3}^{1,1} = 1; X_{5,11}^{1,2} = 1; X_{5,2}^{2,1} = 1; X_{5,10}^{2,2} = 1;$
$X_{5,11}^{2,3} = 1; X_{6,5}^{1,1} = 1; X_{6,1}^{1,2} = 1; X_{6,4}^{2,1} = 1; X_{6,3}^{2,2} = 1; X_{7,2}^{1,1} = 1; X_{7,4}^{1,2} = 1; X_{7,11}^{1,3} = 1; X_{7,4}^{2,1} = 1; X_{7,1}^{2,2} = 1; X_{7,15}^{2,3} = 1; X_{8,10}^{1,1} = 1;$
$X_{8,14}^{1,2} = 1; X_{8,2}^{2,1} = 1; X_{8,10}^{2,2} = 1; X_{8,14}^{2,3} = 1; X_{9,6}^{1,1} = 1; X_{9,7}^{1,2} = 1; X_{9,8}^{1,3} = 1; X_{9,1}^{2,1} = 1; X_{9,6}^{2,2} = 1; X_{9,8}^{2,3} = 1; X_{10,9}^{1,1} = 1; X_{10,4}^{1,2} = 1;$
$X_{10,1}^{1,3} = 1; X_{10,8}^{2,1} = 1; X_{10,6}^{2,2} = 1; X_{10,5}^{2,3}$

or

$Y_1^1 = 1;\ Y_2^2 = 1;\ Y_3^2 = 1;\ Y_4^1 = 1;\ Y_5^1 = 1;\ Y_6^1 = 1;\ Y_7^2 = 1;\ Y_8^1 = 1;\ Y_9^3 = 1;\ Y_{10}^2 = 1;\ Y_{11}^2 = 1;\ Y_{12}^4 = 1;$
$Y_{13}^4 = 1;\ Y_{14}^4 = 1;\ Y_{15}^4 = 1;$
$Z_1^2 = 1;\ Z_2^1 = 1;\ Z_3^1 = 1;\ Z_4^2 = 1;\ Z_5^1 = 1;\ Z_6^1 = 1;\ Z_7^2 = 1;\ Z_8^1 = 1;\ Z_9^2 = 1;\ Z_{10}^2 = 1;$
$X_{1,1}^{1,1} = 1; X_{1,3}^{1,2} = 1; X_{1,10}^{1,3} = 1; X_{1,2}^{2,1} = 1; X_{1,3}^{2,2} = 1; X_{1,13}^{2,3} = 1; X_{2,10}^{1,1} = 1; X_{2,8}^{1,2} = 1; X_{2,15}^{2,1} = 1; X_{2,14}^{2,2} = 1; X_{2,10}^{2,3} = 1;$
$X_{3,2}^{1,1} = 1; X_{3,10}^{1,2} = 1; X_{3,1}^{2,1} = 1; X_{3,11}^{2,2} = 1; X_{4,3}^{1,1} = 1; X_{4,1}^{1,2} = 1; X_{4,14}^{2,1} = 1; X_{4,15}^{2,2} = 1; X_{5,3}^{1,1} = 1; X_{5,11}^{1,2} = 1; X_{5,2}^{2,1} = 1; X_{5,10}^{2,2} = 1;$
$X_{5,11}^{2,3} = 1; X_{6,5}^{1,1} = 1; X_{6,1}^{1,2} = 1; X_{6,4}^{2,1} = 1; X_{6,3}^{2,2} = 1; X_{7,2}^{1,1} = 1; X_{7,4}^{1,2} = 1; X_{7,11}^{1,3} = 1; X_{7,4}^{2,1} = 1; X_{7,1}^{2,2} = 1; X_{7,15}^{2,3} = 1; X_{8,10}^{1,1} = 1;$
$X_{8,14}^{1,2} = 1; X_{8,2}^{2,1} = 1; X_{8,10}^{2,2} = 1; X_{8,14}^{2,3} = 1; X_{9,6}^{1,1} = 1; X_{9,7}^{1,2} = 1; X_{9,8}^{1,3} = 1; X_{9,1}^{2,1} = 1; X_{9,6}^{2,2} = 1; X_{9,8}^{2,3} = 1; X_{10,9}^{1,1} = 1; X_{10,4}^{1,2} = 1;$
$X_{10,1}^{1,3} = 1; X_{10,8}^{2,1} = 1; X_{10,6}^{2,2} = 1; X_{10,5}^{2,3}$

or

$Y_1^1 = 1;\ Y_2^2 = 1;\ Y_3^2 = 1;\ Y_4^1 = 1;\ Y_5^1 = 1;\ Y_6^1 = 1;\ Y_7^2 = 1;\ Y_8^1 = 1;\ Y_9^3 = 1;\ Y_{10}^2 = 1;\ Y_{11}^2 = 1;\ Y_{12}^4 = 1;$
$Y_{13}^4 = 1;\ Y_{14}^5 = 1;\ Y_{15}^5 = 1;$
$Z_1^2 = 1;\ Z_2^1 = 1;\ Z_3^1 = 1;\ Z_4^2 = 1;\ Z_5^1 = 1;\ Z_6^1 = 1;\ Z_7^2 = 1;\ Z_8^1 = 1;\ Z_9^2 = 1;\ Z_{10}^2 = 1;$
$X_{1,1}^{1,1} = 1; X_{1,3}^{1,2} = 1; X_{1,10}^{1,3} = 1; X_{1,2}^{2,1} = 1; X_{1,3}^{2,2} = 1; X_{1,13}^{2,3} = 1; X_{2,10}^{1,1} = 1; X_{2,8}^{1,2} = 1; X_{2,15}^{2,1} = 1; X_{2,14}^{2,2} = 1; X_{2,10}^{2,3} = 1;$
$X_{3,2}^{1,1} = 1; X_{3,10}^{1,2} = 1; X_{3,1}^{2,1} = 1; X_{3,11}^{2,2} = 1; X_{4,3}^{1,1} = 1; X_{4,1}^{1,2} = 1; X_{4,14}^{2,1} = 1; X_{4,15}^{2,2} = 1; X_{5,3}^{1,1} = 1; X_{5,11}^{1,2} = 1; X_{5,2}^{2,1} = 1; X_{5,10}^{2,2} = 1;$
$X_{5,11}^{2,3} = 1; X_{6,5}^{1,1} = 1; X_{6,1}^{1,2} = 1; X_{6,4}^{2,1} = 1; X_{6,3}^{2,2} = 1; X_{7,2}^{1,1} = 1; X_{7,4}^{1,2} = 1; X_{7,11}^{1,3} = 1; X_{7,4}^{2,1} = 1; X_{7,1}^{2,2} = 1; X_{7,15}^{2,3} = 1; X_{8,10}^{1,1} = 1;$
$X_{8,14}^{1,2} = 1; X_{8,2}^{2,1} = 1; X_{8,10}^{2,2} = 1; X_{8,14}^{2,3} = 1; X_{9,6}^{1,1} = 1; X_{9,7}^{1,2} = 1; X_{9,8}^{1,3} = 1; X_{9,1}^{2,1} = 1; X_{9,6}^{2,2} = 1; X_{9,8}^{2,3} = 1; X_{10,9}^{1,1} = 1; X_{10,4}^{1,2} = 1;$
$X_{10,1}^{1,3} = 1; X_{10,8}^{2,1} = 1; X_{10,6}^{2,2} = 1; X_{10,5}^{2,3}$

or

$Y_1^1 = 1;\ Y_2^2 = 1;\ Y_3^2 = 1;\ Y_4^1 = 1;\ Y_5^1 = 1;\ Y_6^1 = 1;\ Y_7^2 = 1;\ Y_8^1 = 1;\ Y_9^3 = 1;\ Y_{10}^2 = 1;\ Y_{11}^2 = 1;\ Y_{12}^4 = 1;$
$Y_{13}^5 = 1;\ Y_{14}^5 = 1;\ Y_{15}^5 = 1;$
$Z_1^2 = 1;\ Z_2^1 = 1;\ Z_3^1 = 1;\ Z_4^2 = 1;\ Z_5^1 = 1;\ Z_6^1 = 1;\ Z_7^2 = 1;\ Z_8^1 = 1;\ Z_9^2 = 1;\ Z_{10}^2 = 1;$
$X_{1,1}^{1,1} = 1; X_{1,3}^{1,2} = 1; X_{1,10}^{1,3} = 1; X_{1,2}^{2,1} = 1; X_{1,3}^{2,2} = 1; X_{1,13}^{2,3} = 1; X_{2,10}^{1,1} = 1; X_{2,8}^{1,2} = 1; X_{2,15}^{2,1} = 1; X_{2,14}^{2,2} = 1; X_{2,10}^{2,3} = 1;$
$X_{3,2}^{1,1} = 1; X_{3,10}^{1,2} = 1; X_{3,1}^{2,1} = 1; X_{3,11}^{2,2} = 1; X_{4,3}^{1,1} = 1; X_{4,1}^{1,2} = 1; X_{4,14}^{2,1} = 1; X_{4,15}^{2,2} = 1; X_{5,3}^{1,1} = 1; X_{5,11}^{1,2} = 1; X_{5,2}^{2,1} = 1; X_{5,10}^{2,2} = 1;$
$X_{5,11}^{2,3} = 1; X_{5,5}^{1,1} = 1; X_{6,1}^{1,2} = 1; X_{6,4}^{2,1} = 1; X_{6,3}^{2,2} = 1; X_{7,2}^{1,1} = 1; X_{7,4}^{1,2} = 1; X_{7,11}^{1,3} = 1; X_{7,4}^{2,1} = 1; X_{7,1}^{2,2} = 1; X_{7,15}^{2,3} = 1; X_{8,10}^{1,1} = 1;$
$X_{8,14}^{1,2} = 1; X_{8,2}^{2,1} = 1; X_{8,10}^{2,2} = 1; X_{8,14}^{2,3} = 1; X_{9,6}^{1,1} = 1; X_{9,7}^{1,2} = 1; X_{9,8}^{1,3} = 1; X_{9,1}^{2,1} = 1; X_{9,6}^{2,2} = 1; X_{9,8}^{2,3} = 1; X_{10,9}^{1,1} = 1; X_{10,4}^{1,2} = 1;$
$X_{10,1}^{1,3} = 1; X_{10,8}^{2,1} = 1; X_{10,6}^{2,2} = 1; X_{10,5}^{2,3}$

Objective Function $F = 5180$
User time for both the models : 0.59 Sec.

A-5 Details of the Inputs for Problems Solved

This section presents the details of inputs for problems solved in Chapter 3

Table A3 : Details of inputs for problems solved

		Processing time per unit; Cost of processing per unit time														
Opn → Part. ↓	Process Plan ↓	1			2			3			4			5		
Part 1 Prod. Vol. (for high range 20; for medium range 10; for low range 10)	1	Machine No.1			Machine No. 3			Machine No. 10			Machine No. 17			Machine No. 18		
		High range	Medi-um range	Low range	High range	Medi-um range	Low range	High range	Medi-um range	Low range	High range	Medi-um range	Low range	High range	Medi-um range	Low range
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1	4;4	2;2	1;1	2;2	1;1	1;1
	2	Machine No. 2			Machine No. 3			Machine No. 13			Machine No. 18					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1	4;4	2;2	1;1			
	3	Machine No. 5			Machine No. 10			Machine No. 14								
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1						
Part 2 Prod. Vol. 24;12;10	1	Machine No. 10			Machine No. 8			Machine No. 14			Machine No. 18					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1	4;4	2;2	1;1			
	2	Machine No. 2			Machine No. 1			Machine No. 10								
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1						
	3	Machine No. 1			Machine No. 14			Machine No. 11								
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1						
Part 3 Prod. Vol. 36;18;10	1	Machine No. 2			Machine No. 10			Machine No. 20								
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1						
	2	Machine No. 1			Machine No. 11			Machine No. 14								
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1						
	3	Machine No. 2			Machine No. 11											
		10;10	5;5	1;1	8;8	4;4	1;1									

Opn → Part. ↓	Process Plan ↓	1			2			3			4		
Part 4	1	Machine No. 3			Machine No. 1			Machine No. 24					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Vol.	2	Machine No. 2			Machine No. 1			Machine No. 28					
40:20:10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 2			Machine No. 1			Machine No. 15					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Part 5	1	Machine No. 3			Machine No. 11			Machine No. 21					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Vol.	2	Machine No. 2			Machine No. 10			Machine No. 11			Machine No. 21		
30:15:10		10;10	5;5	1;1	4;4	2;2	1;1	6;6	3;3	1;1	4;4	2;2	1;1
	3	Machine No. 1			Machine No. 9								
		10;10	5;5	1;1	8;8	4;4	1;1						
Part 6	1	Machine No. 5			Machine No. 1								
Prod.		10;10	5;5	1;1	8;8	4;4	1;1						
Vol.	2	Machine No. 4			Machine No. 3								
22:11:10		10;10	5;5	1;1	8;8	4;4	1;1						
	3	Machine No. 4			Machine No. 9								
		10;10	5;5	1;1	8;8	4;4	1;1						
Part 7	1	Machine No. 2			Machine No. 4			Machine No. 11					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Vol.	2	Machine No. 4			Machine No. 1			Machine No. 2					
38;19;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 1			Machine No. 4			Machine No. 10					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			

Opn → Part. ↓	Process Plan ↓	1			2			3		
Part 8	1	Machine No. 10			Machine No. 1					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1			
Vol.	2	Machine No. 2			Machine No. 10			Machine No. 1		
34;17;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
	3	Machine No. 4			Machine No. 1			Machine No. 9		
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
Part 9	1	Machine No. 6			Machine No. 7			Machine No. 8		
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
Vol.	2	Machine No. 1			Machine No. 6			Machine No. 8		
32;16;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
	3	Machine No. 7			Machine No. 1					
		10;10	5;5	1;1	8;8	4;4	1;1			
Part 10	1	Machine No. 9			Machine No. 4			Machine No. 1		
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
Vol.	2	Machine No. 8			Machine No. 6			Machine No. 5		
36;18;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
	3	Machine No. 3			Machine No. 7			Machine No. 10		
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
Part 11	1	Machine No. 2			Machine No. 8			Machine No. 11		
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
Vol.	2	Machine No. 19			Machine No. 11			Machine No. 8		
48;24;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1
	3	Machine No. 10			Machine No. 5			Machine No. 17		
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1

Opn → Part. ↓	Process Plan ↓	1			2			3			4		
Part 12	1	Machine No. 9			Machine No. 8			Machine No. 23					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Vol.	2	Machine No. 11			Machine No. 9			Machine No. 22					
44;22;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 8			Machine No. 11			Machine No. 25					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Part 13	1	Machine No. 20			Machine No. 21			Machine No. 14					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Vol.	2	Machine No. 13			Machine No. 7			Machine No. 5					
26;13;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 14			Machine No. 15			Machine No. 16					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Part 14	1	Machine No. 6			Machine No. 16			Machine No. 17					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Vol.	2	Machine No. 20			Machine No. 19			Machine No. 18					
38;19;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 20			Machine No. 12			Machine No. 24					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Part 15	1	Machine No. 18			Machine No. 11			Machine No. 28			Machine No. 5		
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1	4;4	2;2	1;1
Vol.	2	Machine No. 20			Machine No. 18			Machine No. 15			Machine No. 1		
34;17;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1	4;4	2;2	1;1
	3	Machine No. 19			Machine No. 15			Machine No. 12			Machine No. 8		
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1	4;4	2;2	1;1

Opn → Part. ↓	Process Plan ↓	1			2			3			4		
Part 16	1	Machine No. 3			Machine No. 13			Machine No. 24			Machine No. 29		
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1	4;4	2;2	1;1
Vol.	2	Machine No. 29			Machine No. 25			Machine No. 20					
36;18;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 27			Machine No. 25			Machine No. 22					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Part 17	1	Machine No. 30			Machine No. 26			Machine No. 28					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Vol.	2	Machine No. 21			Machine No. 27			Machine No. 30					
28;14;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 17			Machine No. 30			Machine No. 7					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Part 18	1	Machine No. 4			Machine No. 16			Machine No. 12			Machine No. 16		
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1	4;4	2;2	1;1
Vol.	2	Machine No. 13			Machine No. 23			Machine No. 22					
42;21;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 26			Machine No. 25			Machine No. 24					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Part 19	1	Machine No. 29			Machine No. 25			Machine No. 22					
Prod.		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
Vol.	2	Machine No. 29			Machine No. 27			Machine No. 16					
32;16;10		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			
	3	Machine No. 27			Machine No. 30			Machine No. 22					
		10;10	5;5	1;1	8;8	4;4	1;1	6;6	3;3	1;1			

Opn → Part. ↓	Process Plan ↓	1			2			3		
Part 20 Prod. Vol. 32;16;10	1	Machine No. 21			Machine No. 19			Machine No. 6		
		10:10	5:5	1:1	8:8	4;4	1;1	6:6	3;3	1;1
	2	Machine No. 26			Machine No. 25			Machine No. 24		
		10:10	5:5	1:1	8;8	4;4	1;1	6;6	3;3	1;1
	3	Machine No. 19			Machine No. 28			Machine No. 23		
		10:10	5:5	1;1	8;8	4;4	1;1	6;6	3;3	1;1

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